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SIMULATION OF HIGH-ORDER HYBRID CONTROL SYSTEMS

PREPARED BY

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FOREWORD

This document is a technical summary of the progress made since January 28, 1967, by the Auburn University Electrical Engineering Department toward fulfillment of phase B of contract No. NAS8-11274. This contract was awarded to Engineering Experiment Station, Auburn, Alabama, May 28, 1964, and was extended September 28, 1966 by the George C. Marshall Space Flight Center, National Aeronautics and Space Administration, Huntsville, Alabama.

SUMMARY

Three distinct methods for simulating a hybrid control system are described in this report. The attitude control system of the Saturn V booster stage was used throughout to illustrate the applicability of these simulation methods. The first simulation procedure uses only analog components with the exception of two sample-and-hold elements. The second method uses both analog and digital devices. The analog computer is used to simulate the continuous-time portion of the system and the digital portion of the system is simulated by a special purpose digital device. Finally, the system is simulated wholly on a digital computer using discrete-time techniques. Each of the three techniques discussed above is shown to result in the development of an accurate simulation for the system model under consideration.

LIST OF PERSONNEL

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LIST OF SYMBOLS

SYMBOL	FORTRAN SYMBOL	DEFINITION
A, $\underline{b}$, $\underline{c}$	A, B, C	Matrices used in the state representation of the continuous part of the system
a_i, b_i	AI, BI	Constants associated with D(z), the digital compensation function
C_1	C1	Aerodynamic moment coefficient
C_2	C2	Control moment coefficient
D(s)		A continuous approximation for D(z), the digital compensation function
D(z)		Digital compensation function
F	FC	Total thrust of the vehicle
G(s)		Transfer function for the continuous part of the system
GC	GC	Control gain
GH*(s)	TOTAL	Pulse transfer function of GH(s)
H(s)	HOL	Transfer function for the zero-order hold
I_E	AIE	Engine moment of inertia about the gimbal point
I_{xx}	AIXX	Moment of inertia about the vehicle center of gravity
k_3	AK3	Control moment coefficient
K_{ij}		Analog computer scale constants

SYMBOL	FORTRAN SYMBOL	DEFINITION
ℓ_{cg}	ALCG	Distance from vehicle center of gravity to the gimbal point
M_i	GM(I)	Generalized mass associated with the i^{th} bending mode
N		The number of encirclements of the (0 db, 180°) point
P		The number of poles of $1 + GH^*(s)$ enclosed by the Nyquist path
R'	RP	Thrust of control engines
S_E	ASE	First moment of swivel about the gimbal point for one engine
T	T	Period of sampler
V	XP	3 X 1 state vector for D(z)
$\underline{x}$	x(k)	State vector for continuous portion of the system
$\dot{\underline{x}}(t)$		$\frac{d}{dt}\underline{x}(t)$
$Y_i(X_\beta)$	YB(I)	Normalized displacement of the i^{th} bending mode at the station of the engine gimbal
$Y_i'(X_\beta)$	YPB(I)	Normalized slope of the i^{th} bending mode at the station of the engine gimbal $= \frac{d}{dX} Y_i(X_\beta)$
$Y_i'(X_D)$	YPD(I)	Normalized slope of i^{th} bending mode at the station of the instrument unit $= \frac{d}{dX} Y_i(X_D)$
$\mathfrak{Z}$		Indicates z-transform operation
$\mathfrak{Z}^{-1}$		Indicates inverse z-transform operation

SYMBOL	FORTRAN SYMBOL	DEFINITION
Z		Number of zeroes of 1 + GH*(s) enclosed by the Nyquist path
α_i		Analog computer scale constants
β		Engine deflection angle
β_c	BETAC	Command engine deflection angle
ζ_i	ZETA(I)	Damping of the i^{th} bending mode
$\zeta 1$	ZETA1	Actuator constant
$\zeta 2$	ZETA2	Actuator constant
η_i		Generalized displacement of the i^{th} bending mode
σ_i		Analog computer scale constant
ϕ	X(6)	Attitude error
Φ	AT	State transition matrix for the continuous portion of the system
ϕ_D		Attitude error at the station of the instrument unit
$\phi_D^*(s)$	PHID	The Laplace transform of the sampled $\phi_D(t)$
ω_i	WB(I)	Frequency of the i^{th} bending mode
$\omega 1$	W1	Actuator constant
$\omega 2$	W2	Actuator constant

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I. INTRODUCTION

In order to design or evaluate the design of a control system, the engineer is almost always concerned with the system's behavior to a specified set of initial conditions or inputs. The purpose of this treatise is to present three different similation techniques which can be used to obtain the time response of a hybrid system to any set of allowable initial conditions. These three techniques can also be used to obtain the response of a hybrid system to any type input provided the input is passed through a zero-order hold. The term "hybrid system" is used here to denote a system in which both continuous-time and discrete-time signals appear.

The hybrid system to be simulated is the Saturn V S1-C thrust vector control system (attitude control system). The differential equations which model the vehicle dynamics during the flight contain some coefficients that vary slowly with time. Since the solution of the given set of differential equations is to be investigated over a relatively short period of time only, the time varying coefficients in these equations will be assumed constant. These differential equations with constant coefficients are used to perform stability and time-domain studies, i.e., the simulation techniques developed in this thesis are applicable for the analysis of stationary linear systems.

The Saturn V S1-C thrust vector control system, which is used to demonstrate the proposed simulation techniques, is described in Chapter II. Specifically, the differential equations which characterize the vehicle, a functional description of the control system, and an analysis of control system stability is presented in this chapter. In Chapter III, a description of hybrid simulation techniques is given and a hybrid simulation development is presented for the Saturn V S1-C system described in Chapter II. Results obtained from this simulation are given and discussed. A method for simulating a hybrid system wholly on a digital computer is developed in Chapter IV. This method is applied to the example system and the results which were obtained from the simulation are discussed.

II. DESCRIPTION OF THE PROBLEM

The following set of equations characterize the Saturn V S1-C vehicle. The symbols are identified in the List of Symbols.

Controller dynamics:

$$\ddddot{\beta} + (2\zeta_1\omega_1 + 2\zeta_2\omega_2)\dddot{\beta} + (\omega_1^2 + 4\zeta_1\zeta_2\omega_1\omega_2 + \omega_2^2)\ddot{\beta} +$$

$$(2\zeta_2\omega_2\omega_1^2 + 2\zeta_1\omega_1\omega_2^2)\dot{\beta} + \omega_1^2\omega_2^2\beta = \omega_1^2\omega_2^2\beta_c \qquad \text{(II-1)}$$

Moment equation:

$$\ddot{\phi} = -C_1\phi - \sum_{i=1}^{4}\left[\frac{F\,\ell_{cg}}{I_{xx}}\,Y_i'(X_\beta) + \frac{F}{I_{xx}}\,Y_i(X_\beta)\right]\eta_i -$$

$$\left(\frac{\ell_{cg}}{I_{xx}}\,S_E + \frac{I_E}{I_{xx}}\right)\ddot{\beta} - \left(\frac{k_3 S_E}{I_{xx}} + C_2\right)\beta \qquad \text{(II-2)}$$

Bending mode equations:

$$\ddot{\eta}_i + 2\zeta_i\,\omega_i\,\dot{\eta}_i + \omega_i^2\,\eta_i = \frac{R'Y_i(X_\beta)}{M_i}\,\beta + \left(\frac{S_E Y_i(X_\beta) - I_E Y_i'(X_\beta)}{M_i}\right)\ddot{\beta}$$

$i = 1, 4$ (II-3)

Attitude error equation:

$$\phi_D = \phi + \sum_{i=1}^{4} Y_i'(X_D)\eta_i \qquad \text{(II-4)}$$

The variable ϕ_D represents the vehicle attitude error as measured by an onboard instrument unit and is the only information available for use in controlling the vehicle. In fact ϕ_D is available only every T seconds; therefore, the system is inherently a sampled-data control system. Further, in the above equations, β denotes the control engine gimbal angle and is the only controllable quantity. The variable β is related to the commanded engine gimbal angle, β_c, by equation (II-1). This system can be represented as shown in Figure 1. An algebraic computation of the G(s) shown in Figure 1 is given in Appendix A.

Without compensation, the vehicle is unstable throughout most of the first stage flight. This instability results because of the $(s^2 + C_1)$ term which appears as a factor of the denominator of G(s) [see Appendix A]. Since C_1 is negative throughout most of the flight, this means that the roots of this factor are located at

$$s_1 = - |C_1|^{\frac{1}{2}} \qquad \text{(II-5)}$$

$$s_2 = + |C_1|^{\frac{1}{2}} \qquad \text{(II-6)}$$

Since ϕ_D is the vehicle attitude error, it is desirable to maintain this variable as near zero as possible. Because of the aforementioned inherent instability, it is manditory that some type of compensating device be included in the system. Since ϕ_D is available as a sampled signal, it is reasonable to consider the use of a

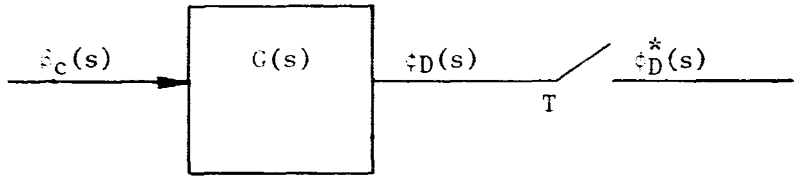


Figure 1. Block Diagram for Open-Loop T.V.C. System

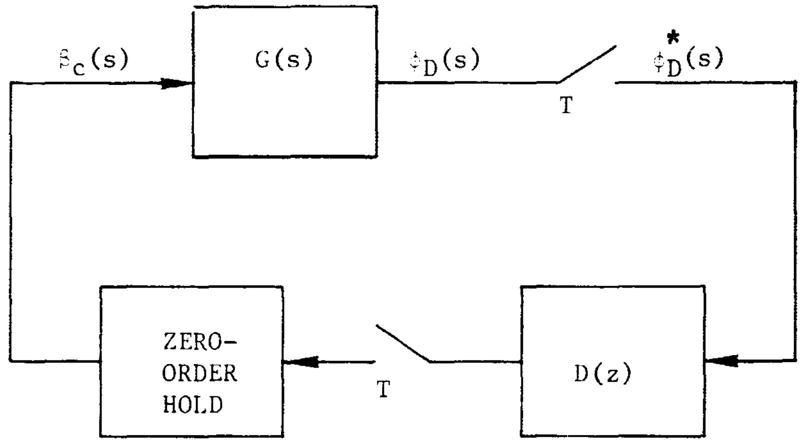


Figure 2. Block Diagram for Closed-Loop T.V.C. System

digital device for compensating the system.

The closed-loop system with a digital compensator whose transfer functi(. is $D(z)$, is shown in Figure 2. The zero-order hold is included to convert the output of the digital device to a continuous-time signal. The transfer function of the zero-order hold is

$$H(s) = \frac{1 - \exp(-Ts)}{s} , \qquad \text{(II-7)}$$

where T is the sampling period. The open loop transfer function of the continuous part of the system is $GH(s)$. The Nyquist diagram for the continuous part of the sampled-data system can be generated by using [1]

$$GH^*(s) = GH(z)\Bigg|_{z = \exp(sT)} = \frac{1}{T} \sum_{\eta=-\infty}^{\infty} GH(s + j\eta\omega_s) \qquad \text{(II-8)}$$

Since $GH(s)$ is low pass with respect to the sampling frequency $\frac{1}{T}$, it is usually adequate to approximate equation (II-8) by the first few terms of the infinite series.

A digital computer program for the calculation of the Nyquist diagram for the uncompensated system using eleven terms of the infinite summation of equation (II-8) is given in Appendix B. The Nyquist diagram for the uncompensated system at 40 seconds of flight is shown in Figure 3.

If a pole of $GH(s)$ is in the right-half s-plane, then that pole will be mapped outside of the unit circle in the z-plane by the z-transformation. Now in order for a closed-loop

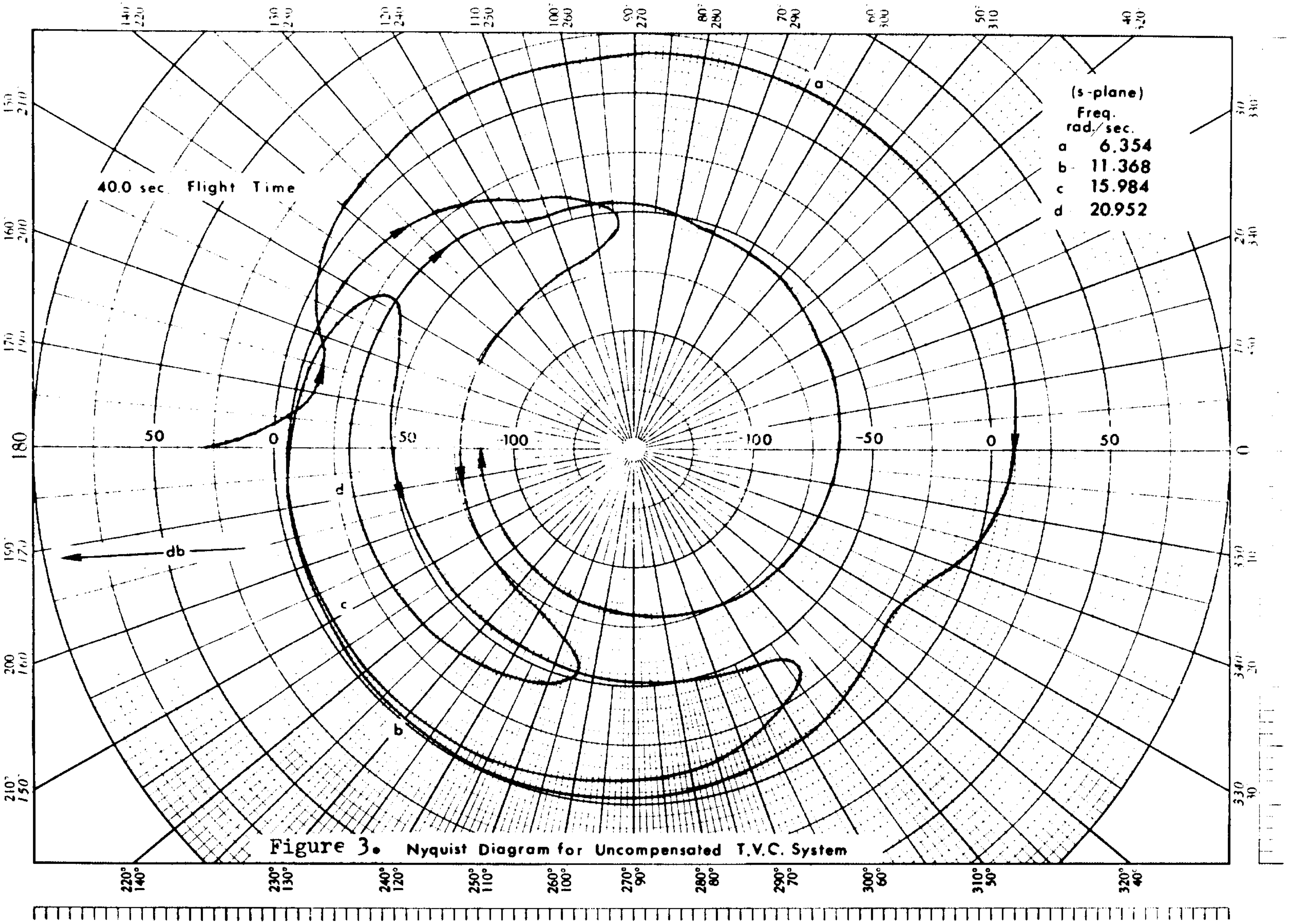


Figure 3. Nyquist Diagram for Uncompensated T.V.C. System

system to be stable, it is necessary that all roots of the characteristics equation lie within the unit circle in the z-plane.

The following statement of the Nyquist Criterion can be used to test for the location of the number of roots, Z, of the characteristic equation, $[1 + GH(z)] = [1 + GH^*(s)]\Big|_{s = \frac{1}{T}\ln(z)} = 0$ which are external to the z-plane unit circle.[1]

Let N = number of encirclements of (0 db, 180^o) point.

Z = number of zeroes of $[1 + GH^*(s)]\Big|_{s = \frac{1}{T}\ln(z)}$ external to the unit circle in the z-plane.

P = number of poles of $[1 + GH^*(s)]\Big|_{s = \frac{1}{T}\ln(z)}$ external to the unit circle.

Then: $N = Z - P.$ (II-9)

The system represented by the Nyquist diagram of Figure 3 is unstable since N = 1. Since, for a closed-loop system, to be stable, Z must be zero, then

$$N = -P. \qquad \text{(II-10)}$$

For the system under consideration P = 1; therefore, N = -1 is necessary in order to achieve stability.

It can be shown that the following digital compensation function stabilizes the proposed closed-loop system. [5]

$$D(z) = 0.4\left[\frac{z^3 - 2.563z^2 + 2.44704z - .877786}{z^3 - 2.38z^2 + 1.886z - .497576}\right] \qquad \text{(II-11)}$$

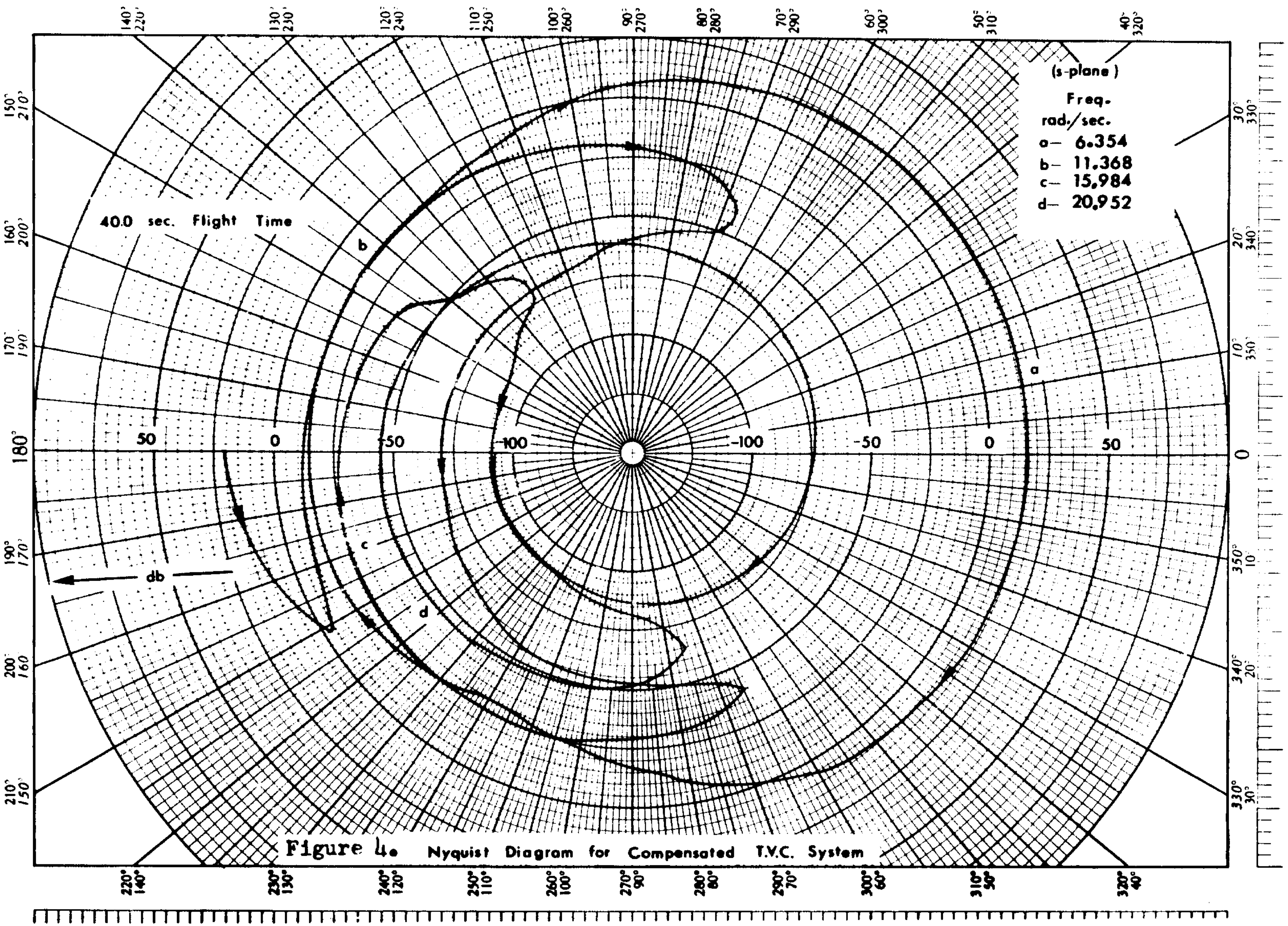


Figure 4. Nyquist Diagram for Compensated T.V.C. System

A digital computer program which incorporates the digital compensation function given by equation (II-11) in the calculation of the Nyquist diagram is given in Appendix C. The compensated Nyquist diagram, using the D(z) given by equation (II-11) and the system dynamics at 40 seconds of flight, is shown in Figure 4. This Nyquist diagram verifies that the closed-loop system is stable, since N = -1.

After a digital compensation function has been proposed, it is necessary to ascertain the time-domain behavior of the resulting system. The next two chapters contain three different approaches to the problem of obtaining a hybrid system's time-domain response to a given set of initial conditions and (or) inputs.

III. ANALOG STUDIES

In this chapter, the problem of hybrid system simulation using analog computer techniques is considered. The system to be simulated is shown in Figure 2. The continuous part of the system, represented by $G(s)$, can be simulated by programming equations (II-1) through (II-4) on the analog computer. The digital portion of the system, $D(z)$, can be simulated by (a) using a continuous-time transfer function in conjunction with two sample and hold elements or, (b) by using a special purpose digital device to realize $D(z)$.

Two factors which must be considered in preparing an analog simulation are (1) time-scale and (2) amplitude-scale. The time-scale problem is directly related to the type of recording equipment to be used in recording results. If the system's operating frequencies lie outside the range of the recording equipment, then the system must be time-scaled. Since the highest frequency of interest in the system of Figure 2 is about 5 hertz, the problem of time-scaling does not arise.

Amplitude-scale factors are equally important. The normal operating range of many computers is $\pm$ 100 volts. The scales for variables in an analog simulation should be chosen such that midrange voltage levels are attained throughout the computer during normal operation. In any case, the signal-to-noise ratio of the problem variables should be maximized.

The problem variables in an analog simulation are related to physical system variables by a conversion constant, i.e., multiplication of each of the problem variables by the appropriate constant changes the units of the problem variables from volts to the units of the physical variable. As an example, suppose that the output of an amplifier is proportional to the physical variable, ϕ, and suppose that the constant of proportionality is 400 volts/radian. Thus when the analog variable measures 40 volts, this corresponds to a system variable, $\phi = 0.1$ radians. Normally the output of this amplifier is written as 400 ϕ on the wiring diagram.

In the simulation of a system, one often knows the maximum magnitude of some of the variables. However, it is usually necessary to estimate the maximum magnitude of many of the system variables in order to develop a satisfactory simulation for the system. Consider the following illustration of these ideas. Equation (II-3) can be written as

$$\ddot{\eta}_i + 2\zeta_i\omega_i\dot{\eta}_i + \omega_i^2\eta_i = K_{1i}\beta + K_{2i}\ddot{\beta} \tag{III-1}$$

It is known that β is physically limited to 0.1 radians. If a step input of $\beta = 0.1$ radians is assumed for (III-1), it turns out that the maximum value of η_i is about 1.0 meter. Consequently a scale factor of 100 volts/meter was chosen for each of the η_i, $i = 1, 2, 3, 4$. In order to satisfactorily amplitude-scale (II-1), the maximum magnitudes of $\dot{\eta}_i$ and $\ddot{\eta}_i$ must be approximated. The maximum magnitudes of these variables can be approximated by [2]

$$\begin{aligned} \dot{\eta}_i(\max) &= \omega_i \eta_i(\max) = \omega_i \\ \ddot{\eta}_i(\max) &= \omega_i \dot{\eta}_i(\max) = \omega_i^2 \end{aligned} \tag{III-2}$$

Equation (II-1) can be written as

$$\ddot{\eta}_i = K_{1i}\beta + K_{2i}\ddot{\beta} - 2\zeta_i\omega_i\eta_i - \omega_i^2\eta_i \ . \tag{III-3}$$

then, (III-3) can be written as

$$\omega_i^2(\ddot{\eta}_i/\omega_i^2) = K_{1i}\beta + K_{2i}\ddot{\beta} - 2\zeta_i\omega_i^2(\eta_i/\omega_i) - \omega_i^2\ \eta_i \tag{III-4}$$

Similarly since $\beta(\max) = 0.1$ radian, the derivative terms can be approximated by

$$\begin{aligned} \dot{\beta}(\max) &= \omega 2\beta(\max) = 0.1\omega 2 \\ \ddot{\beta}(\max) &= \omega 2\dot{\beta}(\max) = 0.1\omega 2^2 \end{aligned} \tag{III-5}$$

Equation (III-4) can now be written as

$$(\ddot{\eta}_i/\omega_i^2) = \left[K_{1i}(0.1)/\omega_i^2\right](\beta/0.1) + \left[K_{2i}(0.1\omega 2^2)/\omega_i^2\right](\ddot{\beta}/0.1\omega 2^2)$$

$$-2\zeta_i\ (\dot{\eta}_i/\omega_i) - \eta_i \qquad \text{(III-6)}$$

Suppose (III-6) is multiplied by E, where E denotes the maximum range of the machine amplifiers in volts.

$$(E\ \ddot{\eta}_i/\omega_i^2) = \left[K_{1i}(0.1)/\omega_i^2\right](\beta E/0.1) + \left[K_{2i}(0.1\omega 2^2)/\omega_i^2\right](\ddot{\beta}E/0.1\omega 2^2)$$

$$-2\zeta_i(\dot{\eta}_i E/\omega_i) - E\eta_i \qquad \text{(III-7)}$$

Let $E\ \ddot{\eta}_i/\omega_i^2 = \ddot{\eta}_i^m$

$$E\ \dot{\eta}_i/\omega_i = \dot{\eta}_i^m$$

$$E\ \eta_i = \eta_i^m \qquad \text{(III-8)}$$

$$E\ddot{\beta}/0.1\omega 2^2 = \ddot{\beta}^m$$

$$E\beta/0.1 = \beta^m$$

where the superscript m denotes a machine variable whose units are volts. Thus (III-7) may be written

$$\ddot{\eta}_i^m = \left[K_{1i}(0.1)/\omega_i^2\right]\beta^m + \left[K_{2i}(0.1\omega 2^2)/\omega_i^2\right]\ddot{\beta}^m - 2\zeta_i\dot{\eta}_i^m - \eta_i^m \qquad \text{(III-9)}$$

Obviously the scale factors relating the magnitude of the physical variables to the magnitude of the machine variables are E/ω_i^2 for $\ddot{\eta}_i$, E/0.1 for β, etc. Note that equation (III-6) is arranged so that each variable is divided by its maximum assumed value. It is desired that when each of these ratios equal unity, the corresponding machine variable will equal 100 volts. The analog simulation of (III-6) is given in Figure 5, where

$$\begin{aligned} \alpha_1 &= K_{1i}(0.1)/\omega_i^2 \\ \alpha_2\alpha_3 &= K_{2i}(0.1\omega 2^2)/\omega_i \\ \alpha_4\alpha_5 &= \omega_i \\ \alpha_6\alpha_7 &= \omega_i \\ \alpha_8 &= 2\zeta_i \end{aligned} \tag{III-10}$$

The analog simulation of equations (II-1) thru (II-4) is shown in Figure 6. Now that a simulation for the continuous portion of the system has been developed, let us next consider the problem of simulating the digital subsystem.

The digital portion of the system, D(z), can be simulated using a continuous approximation for D(z) in conjunction with two sample-hold devices. [1] Consider the realization of D(z) given in Figure 7, where H(s) is given by

$$H(s) = \frac{1 - \exp(-Ts)}{s} \tag{III-11}$$

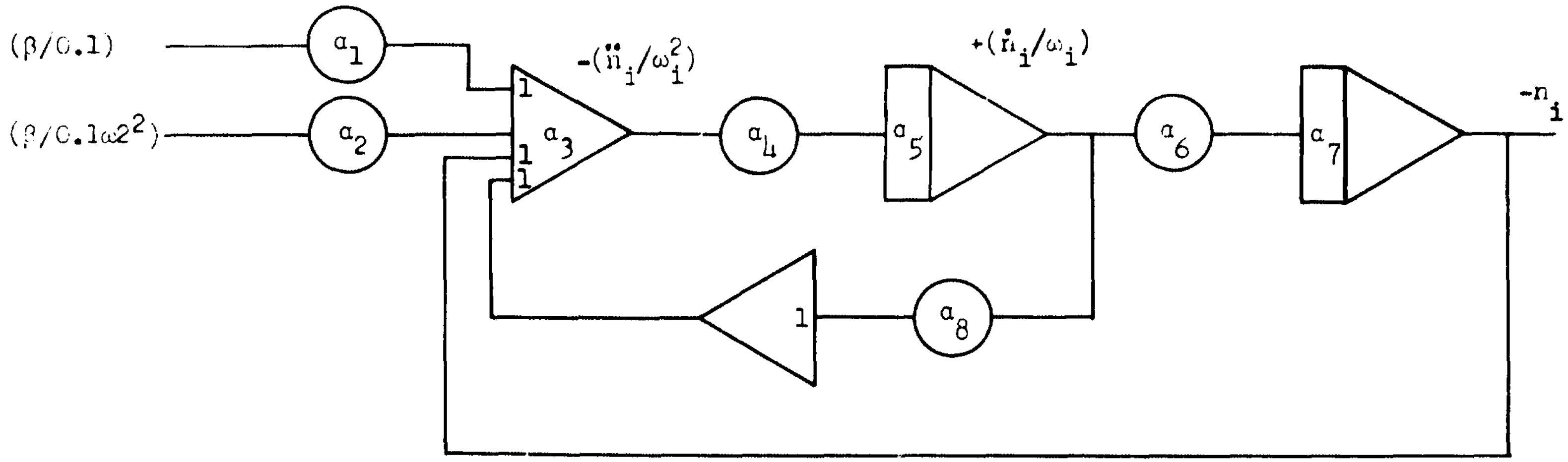


Figure 5. Analog Computer Scaling Used for i th Bending Mode

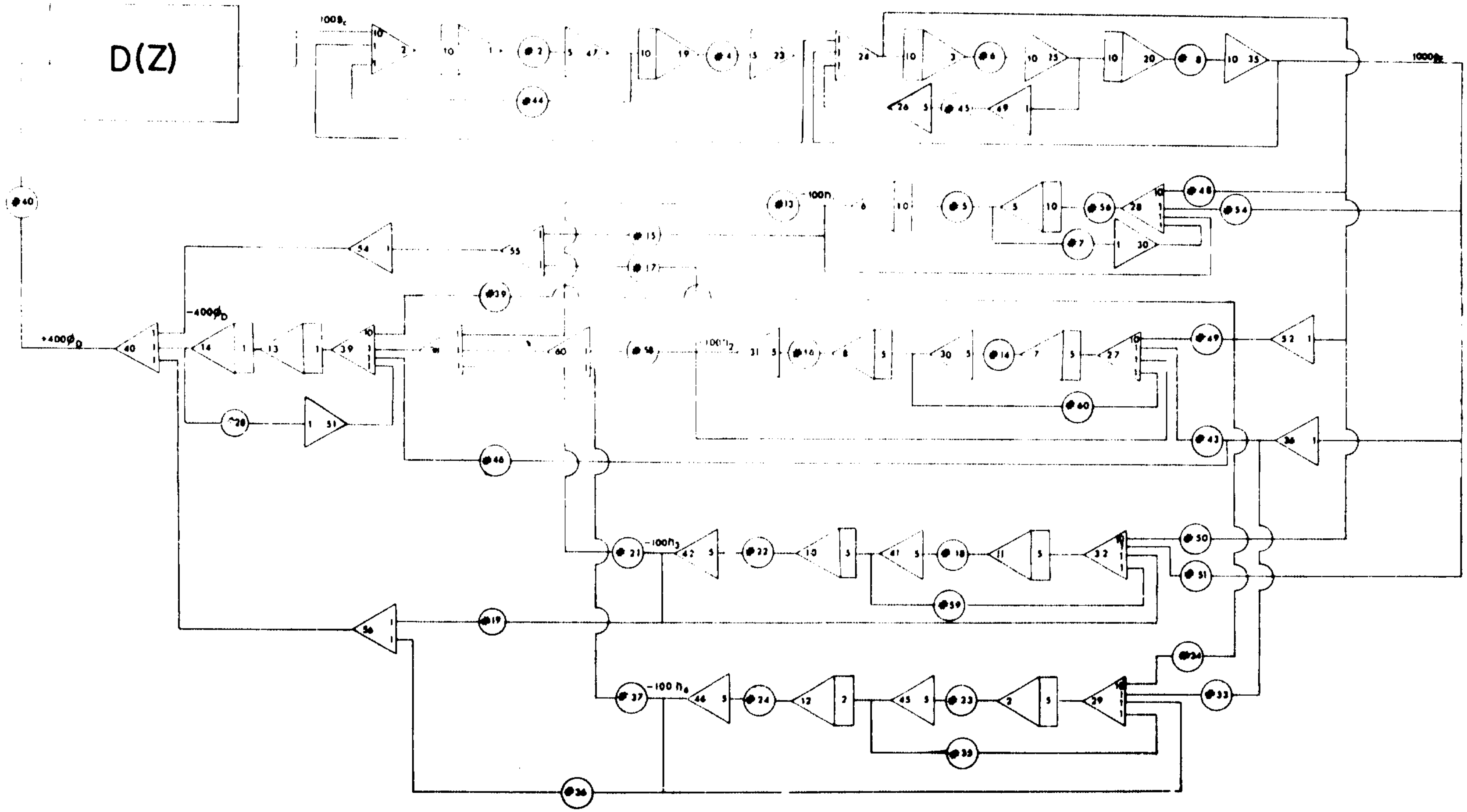


Figure 6. Analog Simulation Diagram of Thrust Vector Control System

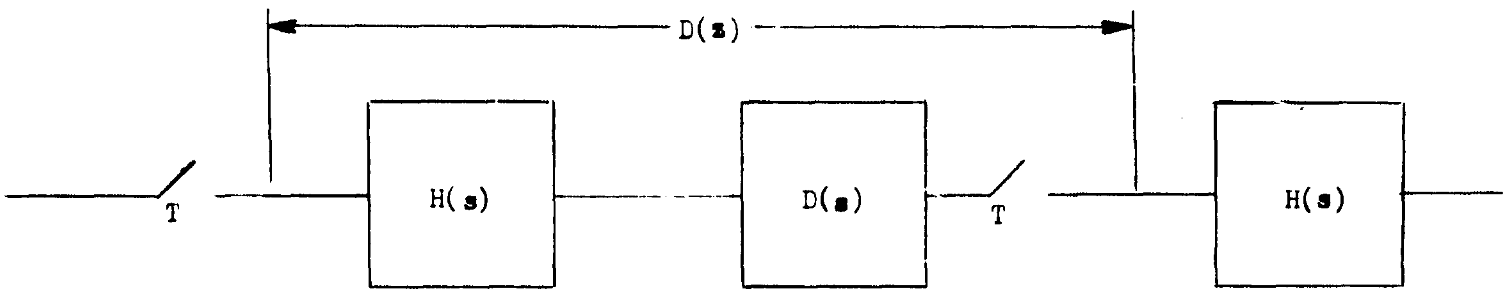


Figure 7. A Realization for D(z) Using Sample and Hold Elements and a Continuous Compensation Function

The problem is to determine that function D(s) with the property that

$$\mathcal{Z}\left[H(s)D(s)\right] = D(z) \tag{III-12}$$

Since H(s) is a zero-order hold, (III-12) becomes

$$D(z) = (1 - z^{-1})\mathcal{Z}\left[\frac{D(s)}{s}\right] \tag{III-13}$$

The solution of (III-13) for D(s)/s is

$$\frac{D(s)}{s} = \mathcal{Z}^{-1}\left[\frac{z}{z-1} D(z)\right] . \tag{III-14}$$

The result of substituting equation (II-11) into (III-14) and performing a partial fraction expansion for the right hand side of (III-14) is

$$\frac{D(s)}{s} = 0.4\mathcal{Z}^{-1}\left\{\frac{.7424z}{(z-1)} + \frac{3.00z}{(z-.82)^2} - \frac{38.9559z}{(z-.82)} + \frac{39.2135z}{(z-.74)}\right\}. \tag{III-15}$$

By utilization of a z-transform table, the inverse operation indicated in (III-15) may be accomplished. Then (III-15) may be solved to give the desired relation for D(s). The result of performing the operations indicated is

$$D(s) = 0.4\left[1.00 - \frac{453.842}{(s+4.961)^2} + \frac{284.748}{s+4.961)} - \frac{295.184}{(s+7.582)}\right] \qquad \text{(III-16)}$$

or in combined form,

$$D(s) = 0.4\left[\frac{s^3 + 7.01s^2 + 272.67s + 137.56}{s^3 + 17.455^2 + 99.31s + 185.29}\right] \qquad \text{(III-17)}$$

The function given by (III-17) was implemented on the analog computer. The step response of D(s),given by (III-17) with the 0.4 gain term omitted, is shown in Figure 8. The step response of D (z), given by (II-11) with the 0.4 gain term omitted, is shown in Figure 9. The step response of D(z) was obtained by recursively solving the difference equations which describe D(z) on a digital computer. The digital computer program used is given in Appendix D. It should be noted that the step response of D(s) very closely follows the step response of D(z) as expected. Theoretically, there should be no difference between these two responses at the sampling instants because a zero-order hold reconstructes the step input signal precisely. However, the differences which do arise are due to imperfections in the analog equipment. The analog computer realization of the D(s) shown in Figure 7 and described by (III-17) except for the 0.4 gain term is shown in Figure 10. In Figure 10,

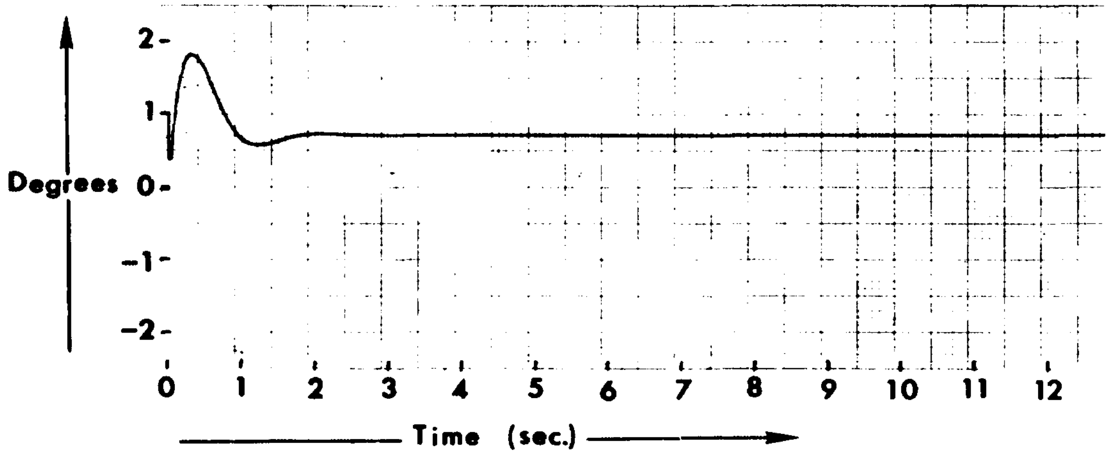


Figure 8. Analog Response of Third-Order D(s) to a Unit Step Input

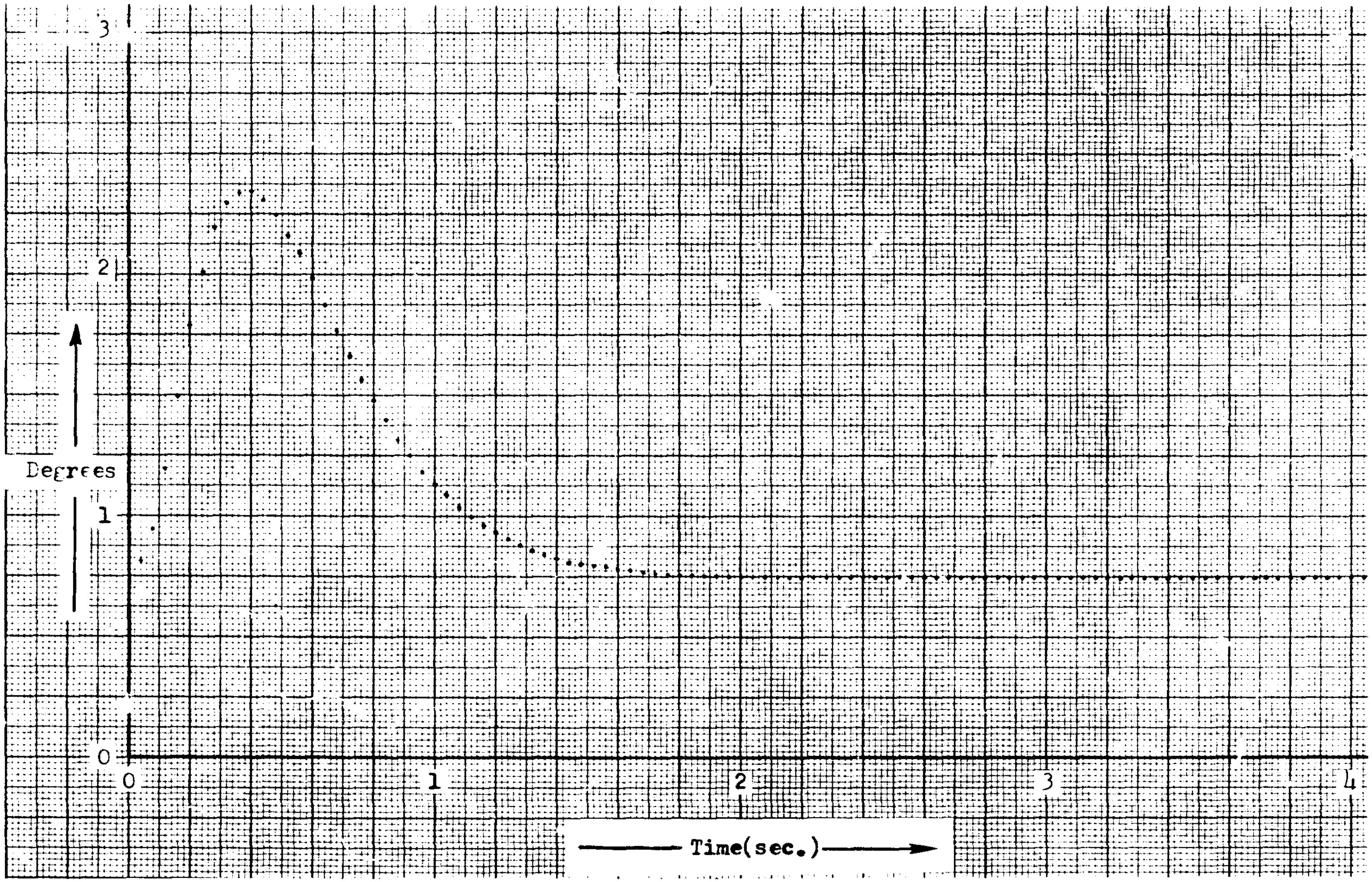


Figure 9. Response of Third-Order D(z) to a Unit Step Input

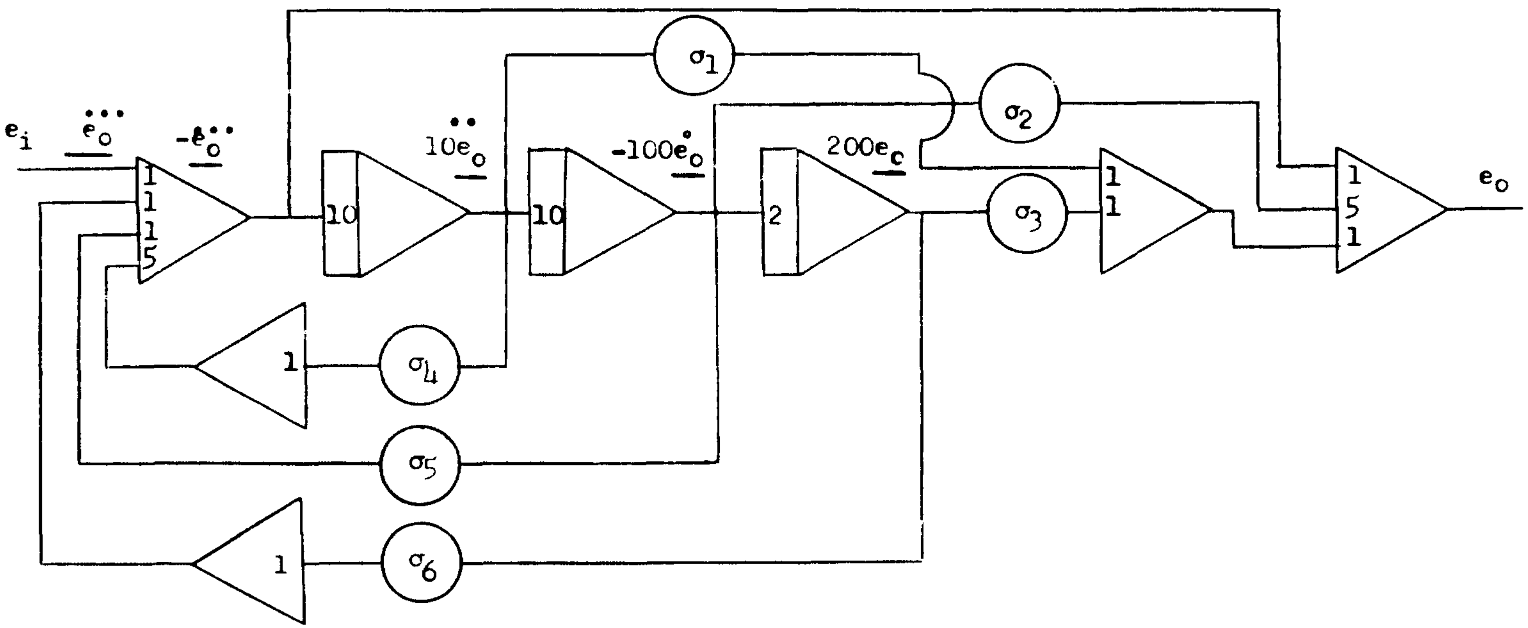


Figure 10. Analog Computer Simulation for D(s)

$$\sigma = 7.01/10$$

$$\sigma = 272.67/500$$

$$\sigma = 137.56/200$$

$$\sigma = 17.45/50 \qquad \text{(III-18)}$$

$$\sigma = 99.307/100$$

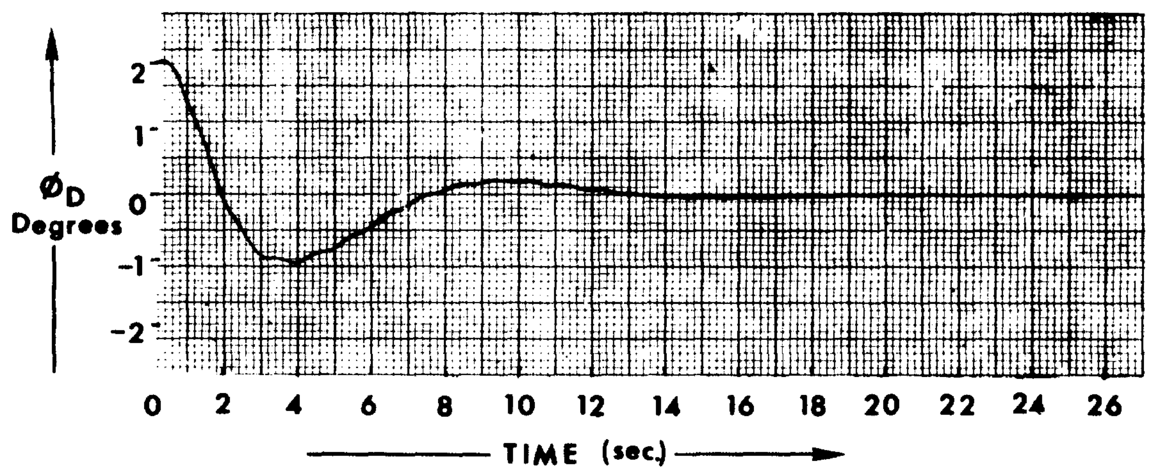
$$\sigma \quad 185.29/200$$

The 0.4 gain term of (III-17) is included in potentiometer number 40 of Figure 6 when the D(s) in conjunction with the two sampler-hold devices is used to simulate D(z). The potentiometer settings of Figure 6 are listed in Appendix H.

Stability studies were performed for the hybrid system using the simulation shown in Figure 6 with D(z) realized by the methods described above. The gain margins checked with the values calculated from the Nyquist diagram shown in Figure 4.

The response of the system at 40 seconds of flight to a 2.0 degree initial condition on ϕ using the D(z) as shown in Figure 7 is given in Figure 11. It was also noted that the response of the given system to a 2.0 degree initial condition on ϕ using only the D(s), with the sampler-hold devices omitted was almost identical to the response shown in Figure 11.

Stability studies were performed on the closed loop system shown in Figure 6 using a special purpose digital device instead of the continuous approximation developed earlier in this chapter. The gain margins checked with the values calculated from the Nyquist



40.0 sec. Flight Time

Figure 11. Analog Response of the T.V.C. System to an Initial Condition on ϕ_D

diagram show 'gure 4. The system response to a 2.0 degree initial condition on ϕ is shown in Figure 12. The difference between the system response shown in Figure 11 and the system response shown in Figure 12 is due to the introdu tion of quantization in the analog-to-digital converter and to the truncation wi hin the digital device. Another potential source of error arose in connection with the analog-to-digital interface. The analog signal had to be scaled down by a factor of 25 in order to insure that input voltage levels did not exceed the maximum dynamic range of the analog-to-digital converter. This requirement may have resulted in a poorer signal to noise ratio, and hence caused some system error. [3]

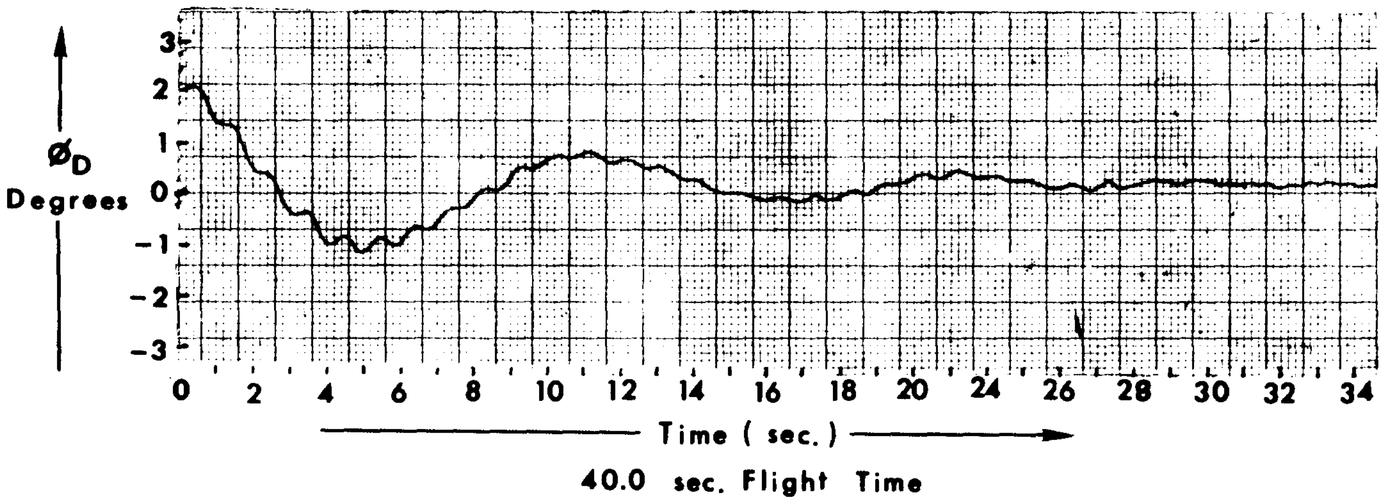


Figure 12. Analog Response of the T.V.C. System to an Initial Condition on ϕ_D Using a Special-Purpose Digital Device

IV. DIGITAL STUDIES

In this chapter, a method will be developed for the digital simulation of a hybrid control system. One method of obtaining a model for a hybrid system is to use discrete-time techniques. Discrete-time techniques are readily applicable for system simulation on a digital computer.

The linear differential equations which describe the Saturn V S1-C at a particular time of flight may be written in the form of first-order differential equations as shown below:

$$\dot{\underline{x}}(t) = A\,\underline{x}(t) + \underline{b}\,\beta_c(t), \tag{IV-1}$$

$$\phi_D(t) = \underline{c}^T\underline{x}(t)\quad , \tag{IV-2}$$

where $\beta_c(t)$ represents commanded engine angle and $\phi_D(t)$ represents the vehicle attitude error as measured at the instrument unit.

Consider the equations of motion given by (II-1) thru (II-4) and the D(z) given by (II-11). A signal flow graph for the system represented by these equations is shown in Figure 13. The a, $\underline{b}$ and $\underline{c}$ matrices of (IV-1) and (IV-2) may be developed for the continuous part of the system shown in Figure 13. The elements of these matrices are given in Appendix E.

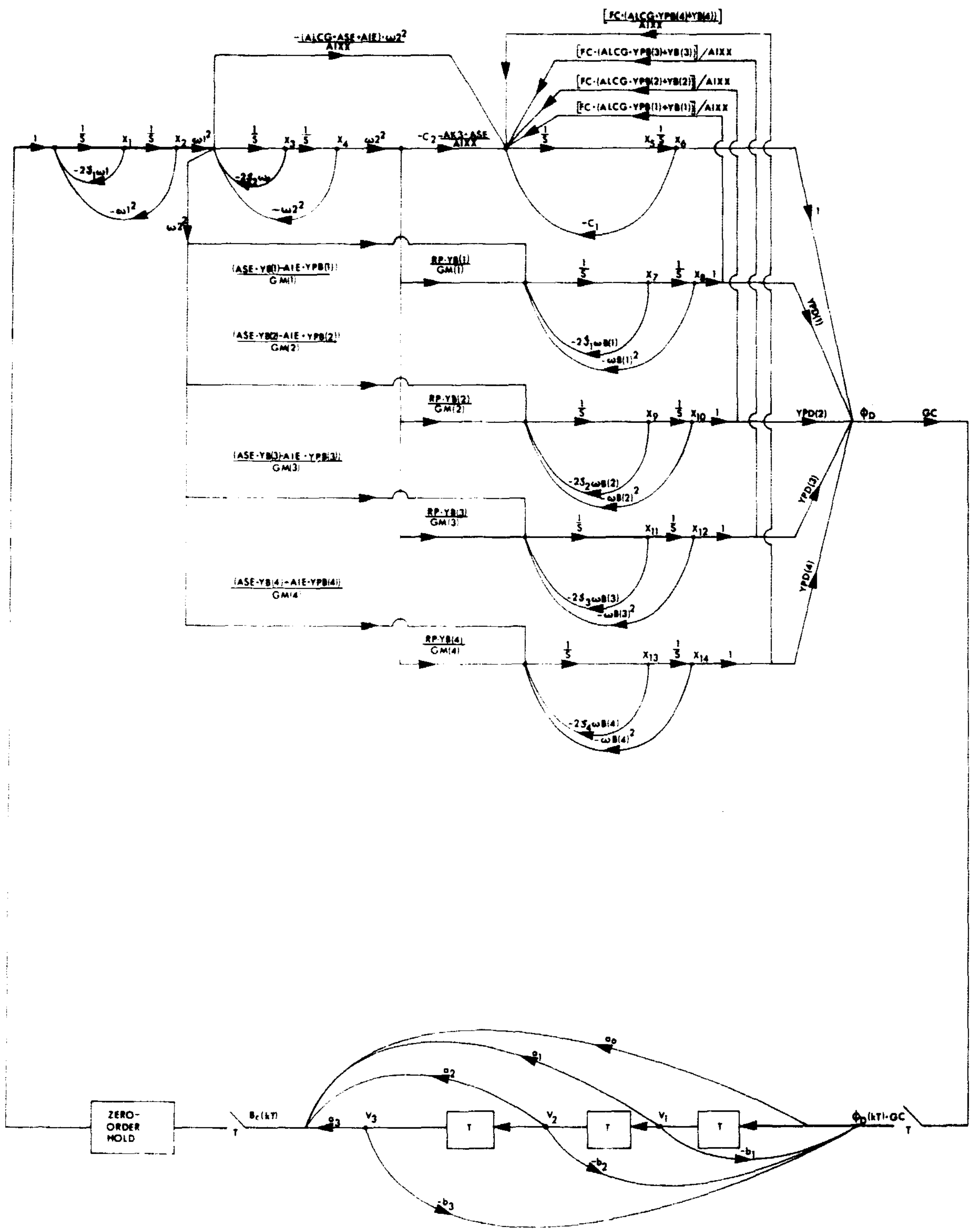


Figure 13. Signal Flow Graph for T.V.C. System

The solution of equation (IV-1) is: [4]

$$\underline{x}(t) = \Phi(t - t_0)\,\underline{x}(t_0) + \int_{t_0}^{t} \Phi(t - \tau)\underline{b}\;\beta_c(\tau)d\tau, \tag{IV-3}$$

where $\Phi(t - t_0)$ is a square matrix given by:

$$\Phi(t - t_o) = \exp\left[A(t - t_0)\right] = \sum_{k=0}^{\infty} \frac{A^k(t - t_0)^k}{k!} \tag{IV-4}$$

The matrix $\Phi(t - t_0)$ is called the state transition matrix since it maps an initial state $\underline{x}(t_0)$ into the state $\underline{x}(t)$ at any later time $t \geq t_0$.

Now, the output of the system, $\phi_D(t)$, is measured only every T seconds, where T is the system sampling period. Further, since a digital controller is used, the engine command single, $\beta_c(t)$, can change only every T seconds and must remain constant between sampling instants. Using this information, the solution of the state equations can be discretized in the following manner. A property of the state transition matrix is:

$$\int_{\tau=0}^{T} \Phi(T - \tau)d\tau = \int_{\tau=0}^{T} \Phi(\tau)d\tau \tag{IV-5}$$

Proof: Let $\sigma = T - \tau$, then $d\sigma = -d\tau$ and

$$\int_{\tau=0}^{T} \Phi(T - \tau)d\tau = \int_{\sigma=T}^{0} \Phi(\sigma)\quad(-d\sigma) = \int_{\sigma=0}^{T} \Phi(\sigma)d\sigma \tag{IV-6}$$

Now, suppose that the system state $\underline{x}(kT)$ and $\beta_c(kT)$ are known and it is desired to determine the state, $\underline{x}[(k+1)T]$. From (IV-3) it is evident that

$$\underline{x}[(k+1)T] = \Phi[(k+1)T - (kT)]\,\underline{x}(kT) + \int_{\tau=kT}^{(k+1)T} \Phi[(k+1)T - \tau]\underline{b}\beta_c(\tau)d\tau. \quad \text{(IV-7)}$$

Note that $\beta_c(\tau) = \beta_c(kT)$ for $kT \leq \tau < (k+1)T$ since $\beta_c(\tau)$ is the output of a zero-order hold. Thus (IV-7) becomes

$$\underline{x}[(k+1)T] = \Phi(T)\,\underline{x}(kT) + \left[\int_{\tau=kT}^{(k+1)T} \Phi[(k+1)T - \tau]d\tau\right]\underline{b}\beta_c(kT). \quad \text{(IV-8)}$$

Using the property that

$$\int_{\tau=0}^{t} f(\tau)d\tau = \int_{\tau=\Delta}^{t+\Delta} f(\tau - \Delta)d\tau, \quad \text{(IV-9)}$$

and that

$$\Phi(t_1 + t_2) = \Phi(t_1)\,\Phi(t_2), \quad \text{(IV-10)}$$

we can write

$$\int_{\tau=kT}^{(k+1)T} \Phi[(k+1)T - \tau]d\tau = \Phi(T)\int_{\tau=kT}^{(k+1)T} \Phi\left[-(\tau - kT)\right]d\tau = \Phi(T)\int_{\tau=0}^{T} \Phi(-\tau)d\tau$$

$$= \int_{\tau=0}^{T} \Phi(T - \tau)d\tau \quad \text{(IV-11)}$$

Then from (IV-5) and (IV-11), (IV-8) can be expressed as

$$\underline{x}\,[(k+1)T\,] = \Phi(T)\,\underline{x}(kT) + \left[\int_{\tau=0}^{T} \Phi(\tau)d\tau\right] \underline{b}\ \beta_c(kT). \qquad \text{(IV-12)}$$

The matrix $\Phi(T)$ and the matrix $\int_{\tau=0}^{T} \Phi(\tau)d\tau$ are constant matrices which need be evaluated only once. The Taylor Series expansion of these matrix functions can be used to obtain numerical results for the matrix coefficients. The expansions are:

$$\Phi(T) = I + AT + \frac{A^2T^2}{2!} + \frac{A^3T^3}{3!} + \ldots, \qquad \text{(IV-13)}$$

and ,

$$\int_0^T \Phi(\tau)\, d\tau = IT + \frac{AT^2}{2!} + \frac{A^2T^3}{3!} + \frac{A^3T^4}{4!} + \ldots\ . \qquad \text{(IV-14)}$$

It was found for the system under consideration that $\Phi(T)$ and $\int_0^T \Phi(\tau)d\tau$ can be accurately computed by using 25 terms of (IV-13) and (IV-14) respectively. Let $A_1 = \Phi(T)$, and $\underline{b}_1 = \left[\int_0^T \Phi(\tau)d\tau\ \underline{b}\right]$. Therefore from (IV-12) and (IV-2)

$$\underline{x}\,[(k+1)T] = A_1\,\underline{x}(kT) + \underline{b}_1\,\beta_c(kT) \qquad \text{(IV-15)}$$

$$\phi_D(kT) = \underline{c}^T\underline{x}(kT) \qquad \text{(IV-16)}$$

Equations (IV-15) and (IV-16) represent the discretized system dynamics. The closed loop system can now be represented by the block diagram shown in Figure 14.

The digital compensation function of (II-11) may be written as

$$D(z) = \frac{\beta_c(z)}{\phi_D(z)} = GC\left[\frac{z^3 - 2.563z^2 + 2.44704z - .877786}{z^3 - 2.38z + 1.886z - .497576}\right], \quad \text{(IV-17)}$$

where

$$GC = 0.4 \quad .$$

Rewriting (IV-17) ,

$$D(z) = GC\left[\frac{a_0z^3 + a_1z^2 + a_2z + a_3}{z^3 + b_1z^2 + b_2z + b_3}\right] \quad \text{(IV-18)}$$

where $a_0 = 1.0$, $a_1 = -2.563$, $a_2 = 2.44704$, $a_3 = -.877786$,

$b_1 = -2.38$, $b_2 = 1.886$, and $b_3 = -.497576$.

This transfer function represents a system which is characterized by difference equations. One set of first-order difference equations whose transfer function is (IV-18) is:

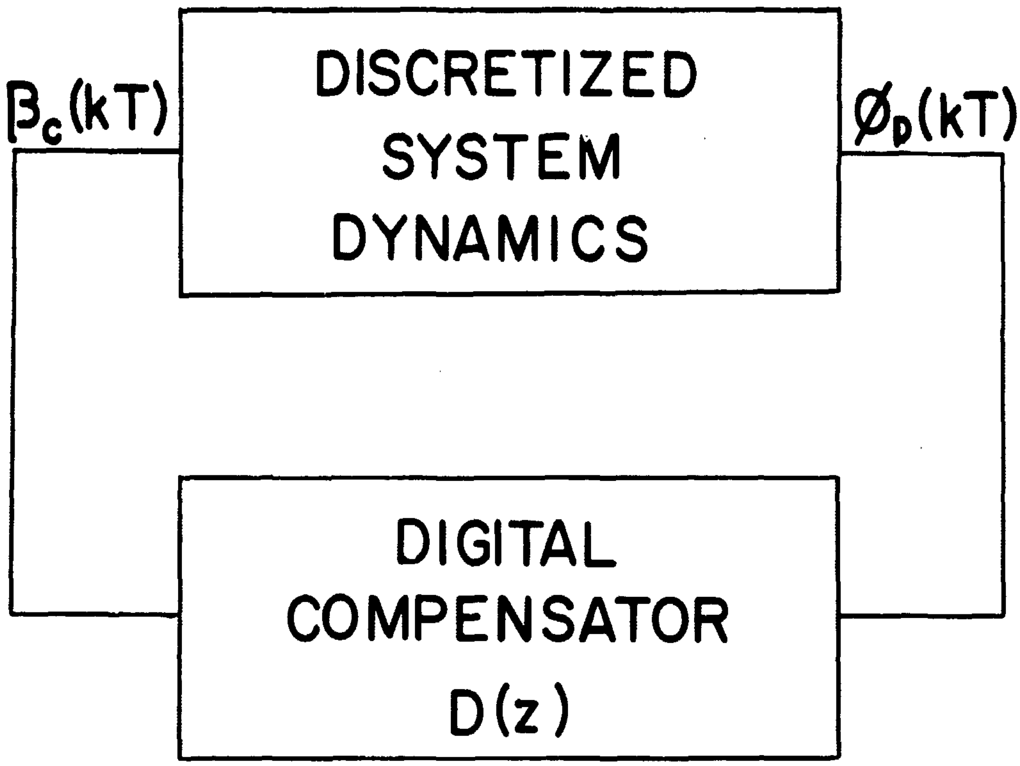


Figure 14. Block Diagram of Discretized System

$$V_1[(k+1)T] = -b_1V_1(kT) - b_3V_3(kT) + \Phi_D(kT) \cdot GC - b_2v_2(kT)$$
$$V_2[(k+1)T] = V_1(kT) \qquad \text{(IV-19)}$$
$$V_3[(k+1)T] = V_2(kT)$$

$$\beta_c(kT) = (a_1 - b_1 \cdot a_0)\ V_1(kT) + (a_2 - b_2 \cdot a_0)\ V_2(kT)$$
$$+ (a_3 - b_3 \cdot a_0)\ V_3(kT) + \Phi_D(kT) \cdot GC \cdot a_0 \qquad \text{(IV-20)}$$

Equations (IV-19) can be mechanized as shown by Figure 15.

The calculation of the response of the system to a given set of initial condition is accomplished as follows: vehicle initial states, $\underline{x}(0)$, and compensator initial states, $\underline{V}(0)$, are chosen. Then $\Phi_D(0)$ and $\beta_c(0)$ may be calculated by (IV-2) and (IV-20) respectively. Next, (IV-15) and (IV-19) are used to calculate $\underline{x}(T)$ and $\underline{V}(T)$. Then $\Phi_D(T)$ and $\beta_c(T)$ are computed. The process is repeated for all kT of interest.

The digital computer program given in Appendix F was used to implement the above steps. A time response of the system under consideration using the vehicle data at 40 seconds of flight is given in Figure 16.

The approach discussed above is particularly useful in studying the time domain behavior of the attitude control system when considering the effects of quantization in the implementation of D(z). The digital computer subroutine given in Appendix G is programmed to simulate the quantization of the actual digital device used in

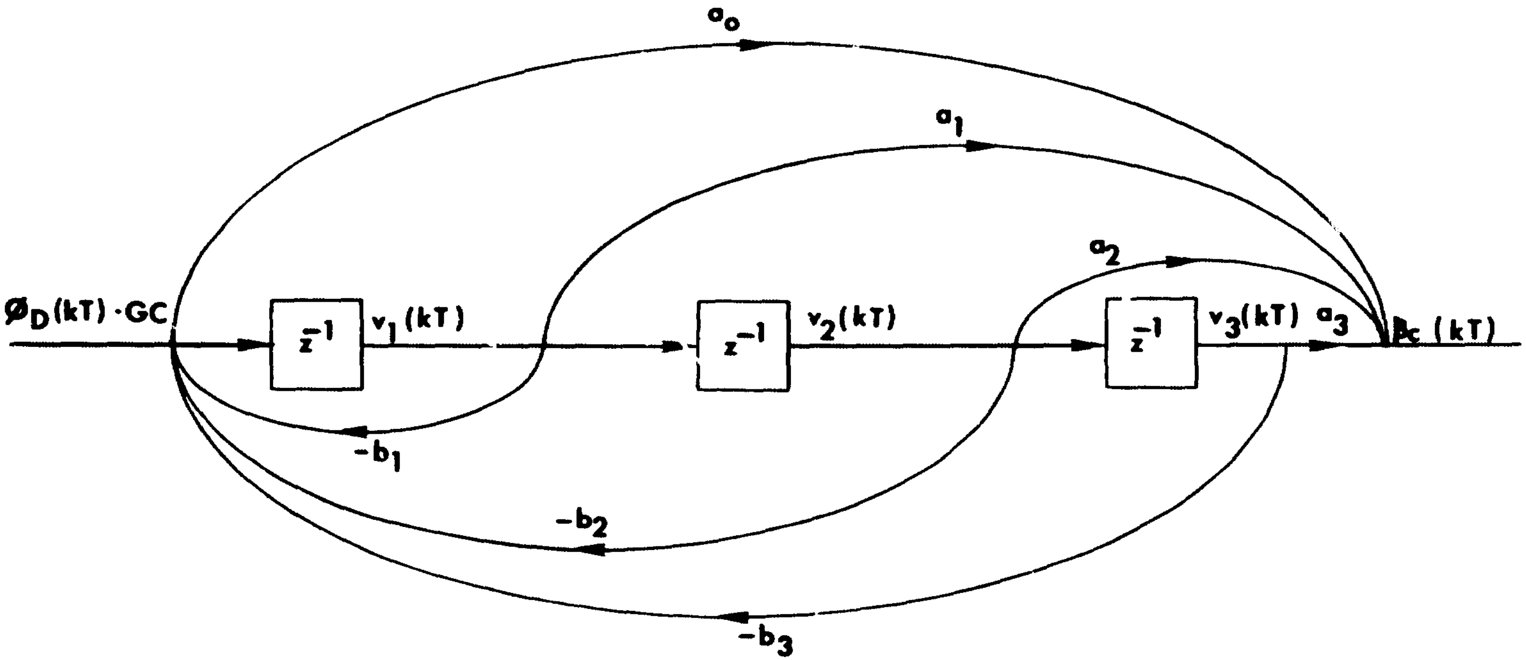


Figure 15. A Realization of D(z)

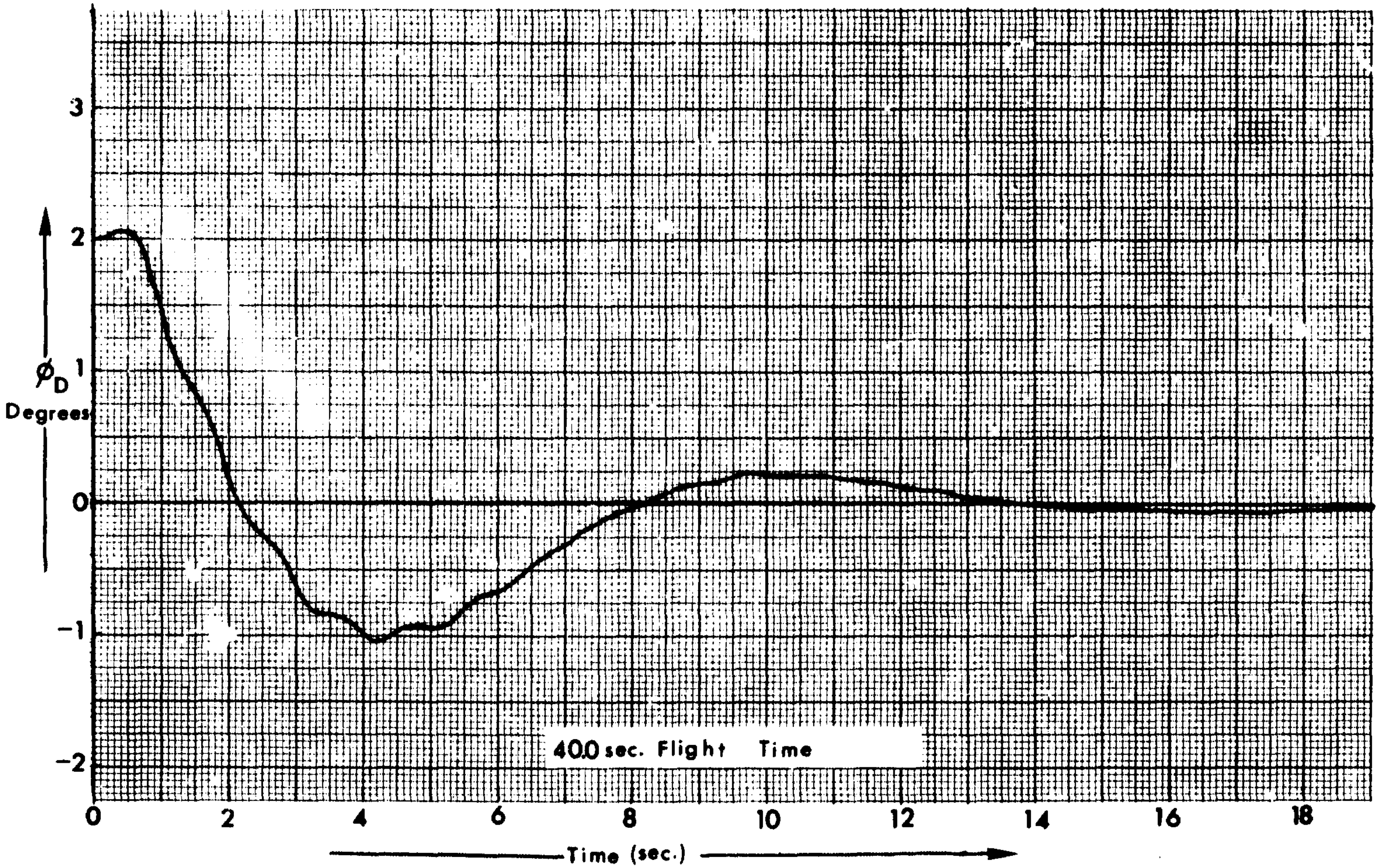


Figure 16. Response of the T.V.C. System to an Initial Condition on ϕ_D

Chapter III for D(z). [3] The subroutine is programmed for a maximum of 15 degrees input to the D(z) and there exists 255 discrete levels in the analog to digital converter. The truncation of signals and coefficient quantization is programmed in exactly the same manner as they actually occur in the digital compensation used in Chapter III used to generate the system response shown in Figure 12. The system response to a 2.0 degree initial condition on ϕ calculated using the digital computer simulation modified to include quantization is shown in Figure 17. Note that the response shown in Figure 17 is in almost complete agreement with the response shown in Figure 12 which was obtained by using the special purpose digital device in conjunction with the analog simulation.

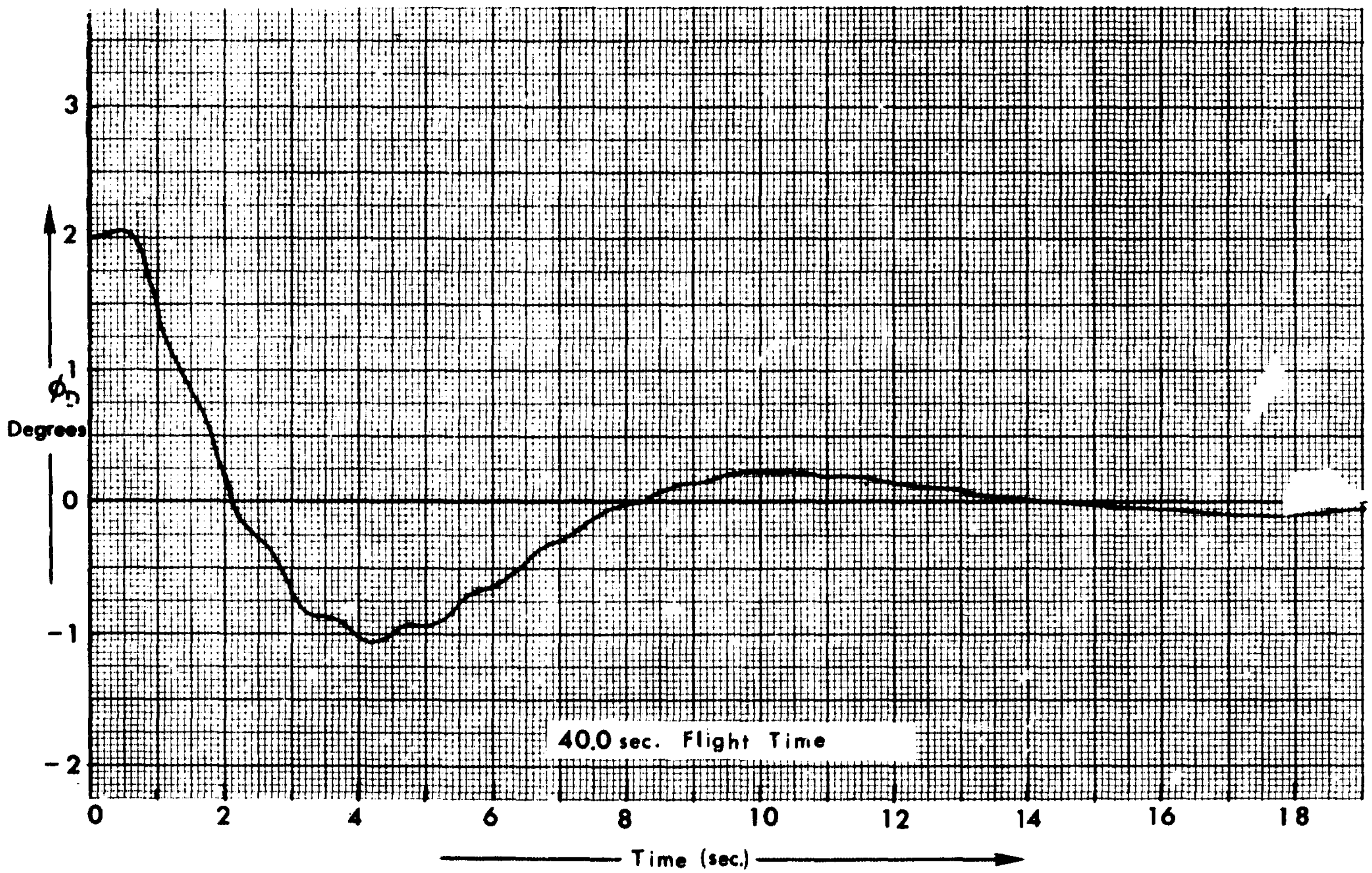


Figure 17. Response of the T.V.C. System to an Initial Condition on ϕ_D with Quantization Included

V. CONCLUSIONS

In this report, three distinct methods for simulating hybrid control systems are described. The attitude control system of a large space vehicle was used throughout to illustrate the applicability of these simulation methods.

The first simulation procedure uses, with the exception of two sample-and-hold elements, only analog components. The next method of simulation uses both analog and digital devices. The continuous-time portion of the system is simulated on an analog computer and the digital portion of the system was simulated by a special purpose digital device. Finally, the system was simulated on a digital computer using discrete-time techniques.

Each of these three techniques provided almost identical time responses. Further, each of these simulations was used to confirm the stability margins established analytically in Chapter II for the system under consideration. The sources of possible error and the limitations of each simulation method was outlined in each section.

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APPENDICES

APPENDIX A

AN ALGEBRAIC COMPUTATION OF $\phi_D(s)/\beta_c(s)$

The Laplace Transformation of equations (II-1, 4) yields:

$$\frac{\beta}{\beta_c} = \frac{\omega 1^2 \omega 2^2}{(s^2 + 2\zeta 1\omega 1 s + \omega 1^2)(s^2 + 2\zeta 2\omega 2 s + \omega 2^2)} \qquad \text{(A-1)}$$

$$s^2\phi = -C_1\phi - \sum_{i=1}^{4} \left[\frac{F\,\ell_{cg}}{I_{xx}} Y_i'(X_\beta) + \frac{F}{I_{xx}} Y_i(X_\beta) \right] \eta_i - \left[\left(\frac{\ell_{cg} S_E}{I_{xx}} + \frac{I_E}{I_{xx}} \right) s^2 + \left(\frac{k_3 S_E}{I_{xx}} - C_2 \right) \right] \beta \qquad \text{(A-2)}$$

$$(s^2 + 2\zeta_i\omega_i s + \omega_i^2)\eta_i = \frac{1}{M_i} \Big[R'Y_i(X_\beta) + (S_E Y_i(X_\beta) - I_E Y_i'(X_\beta))s^2 \Big] \beta, \quad i = 1,4 \qquad \text{(A-3)}$$

$$\phi_D = \phi + \sum_{i=1}^{4} Y_i'(X_D)\eta_i \qquad \text{(A-4)}$$

From (A-3) we have:

$$\eta_i = \frac{\frac{1}{M_i}\Big[R'Y_i(X_\beta) + (S_E Y_i(X_\beta) - I_E Y_i'(X_\beta))s^2 \Big]}{s^2 + 2\zeta_i\omega_i s + \omega_i^2} \beta \qquad \text{(A-5)}$$

$i = 1, 4$ or,

$$\eta_i = F_i \beta \qquad \text{(A-6)}$$

From (A-4) we have

$$\phi = \phi_D - \sum_{i=1}^{4} Y_i'(X_D)\eta_i \tag{A-7}$$

The substitution of (A-6) into (A-7) yields:

$$\phi = \phi_D - \sum_{i=1}^{4} Y_i'(X_D)F_i\beta \tag{A-8}$$

The substitution of (A-8), and (A-6) into (A-2) yields:

$$s^2\left[\phi_D - \sum_{i=1}^{4} Y_i'(X_D)F_i\beta\right] = -C_1\left[\phi_D - \sum_{i=1}^{4} Y_i'(X_D)F_i\beta\right]$$
$$-\sum_{i=1}^{4}\left[\frac{F\ \ell cg}{I_{xx}} Y_i'(X_\beta) + \frac{F}{I_{xx}} Y_i(X_\beta)\right] F_i\beta$$
$$-\left[\left(\frac{\ell_{cg}S_E}{I_{xx}} + \frac{I_E}{I_{xx}}\right) s^2 + \left(\frac{k3S_E}{I_{xx}} - C_2\right)\right]\beta \tag{A-9}$$

Simplifying, we obtain

$$\phi_D[s^2 + C_1] = \beta\left\{\sum_{i=1}^{4} Y_i'(X_D)F_i\,[s^2 + C_1]\right.$$
$$-\sum_{i=1}^{4}\left[\frac{F\ell_{cg}}{I_{xx}} Y_i'(X_\beta) + \frac{F}{I_{xx}} Y_i(X_\beta)\right]F_i \tag{A-10}$$
$$\left.-\left[\left(\frac{\ell_{cg}S_E}{I_{xx}} + \frac{I_E}{I_{xx}}\right)s^2 + \left(\frac{k_3S_E}{I_{xx}} - C_2\right)\right\}\right. ,$$

or

$$\phi_D \, G_1(s) = \beta \, G_2(s) \; . \tag{A-11}$$

From (A-1) we have

$$\beta = G_3(s) \, \beta_c \; . \tag{A-12}$$

The substitution of (A-12) into (A-11) yields:

$$\phi_D \, G_1(s) = G_3(s) \, \beta_c \, G_2(s) \quad , \tag{A-13}$$

or,

$$\phi_D / \beta_c = G_2(s) \, G_3(s) / G_1(s) = G(s), \tag{A-14}$$

where:

$$G_1(s) = s^2 + C_1 \tag{A-15}$$

$$G_2(s) = \sum_{i=1}^{4} Y_i'(X_D) F_i \left[s^2 + C_1 \right]$$

$$- \sum \left[\frac{F \ell_{cg}}{I_{xx}} \, Y_i'(X_\beta) + \frac{F}{I_{xx}} \, Y_i(X_\beta) \right] F_i$$

$$-\left[\left(\frac{\ell_{cg}S_E}{I_{xx}} + \frac{I_E}{I_{xx}}\right)s^2 + \left(\frac{k_3S_E}{I_{xx}} - C_2\right)\right] \tag{A-16}$$

$$G_3(s) = \omega 1^2\omega 2^2/(s^2 + 2\zeta 1\omega 1s + \omega 1^2)(s^2 + 2\zeta 2\omega 2s + \omega 2^2) \tag{A-17}$$

APPENDIX B

UNCOMPENSATED NYQUIST

```
C     COMPUTATION OF PHID/BETAC SAMPLED FREQUENCY RESPONSE
C
      DIMENSION WB(4),ZETA(4),GM(4),YB(4),YPB(4),YPD(4),
     1 U1(4),U2(4),A(4),B(4)
      COMPLEX CMPLX,CEXP,S,WSS,HOL,TF,GCOMP,WP,Z,S1,SUMAF,
     1 DEN,ANUM,TOTAL,SQ,AF(4),AD,AN,SUMBAF,SUMDAF
   10 FORMAT(3(5X,E15.8))
   11 FORMAT(4(5X,E15.8))
   50 CONTINUE
      READ(5,10)C1,C2,FC
      READ(5,10)ASE,AK3,RP
      READ(5,11)ALCG,AIXX,AIE,TIME
      READ(5,11)WB(1), WB(2), WB(3), WB(4)
      READ(5,11)ZETA(1), ZETA(2), ZETA(3), ZETA(4)
      READ(5,11)GM(1), GM(2), GM(3), GM(4)
      READ(5,11)YB(1), YB(2), YB(3), YB(4)
      READ(5,11)YPB(1), YPB(2), YPB(3), YPB(4)
      READ(5,11)YPD(1), YPD(2), YPD(3), YPD(4)
      DO 9 I=1,4
    9 WB(I)=WB(I)*2.0*3.1415927
   33 FORMAT(1H1,4X,10HINPUT DATA,//)
      AK3 = AK3*57.29578
      PRINT 33
      PRINT 10,C1,C2,FC
      PRINT 10,ASE,AK3,RP
      PRINT 11,ALCG,AIXX,AIE,TIME
      PRINT 11, WB(1), WB(2), WB(3), WB(4)
      PRINT 11, ZETA(1), ZETA(2), ZETA(3), ZETA(4)
      PRINT 11, GM(1), GM(2), GM(3), GM(4)
      PRINT 11, YB(1), YB(2), YB(3), YB(4)
      PRINT 11, YPB(1), YPB(2), YPB(3), YPB(4)
      PRINT 11, YPD(1), YPD(2), YPD(3), YPD(4)
      ALPHA = (ALCG*ASE+AIE)/AIXX
      GAMMA=AK3*ASE/AIXX
      BETA=FC*ALCG/AIXX
      DO 501 I=1,4
      A(I)=RP*YB(I)/GM(I)
      B(I)=(ASE*YB(I)-AIE*YPB(I))/GM(I)
      U1(I)=2.0*ZETA(I)*WB(I)
  501 U2(I)=WB(I)**2
C     CALCULATE FREQUENCY RESPONSE
      DEL = 0.2
      XK = 1.0
   66 FORMAT(1H1,4X,30HFREQUENCY RESPONSE OUTPUT DATA ,//)
      PRINT 66
      OMEG = .005
```

```
      GO TO 4
   77 DEL = DEL/2.0
      OMEG = OMEG-DEL
    4 TF = CMPLX(0.0, 0.0)
      TS= 0.04
      OMEGS = 2.0*3.1415927/TS
      NTILT = 5
      NX = 2*NTILT+1
      DO 24 J = 1, NX
      XJ = -NTILT+J-1
      OMEG1 = OMEG+XJ*OMEGS
      S = CMPLX(0.0, OMEG1)
      SQ=S**2
      DO 502 I=1,4
  502 AF(I)=(A(I)+B(I)*SQ)/(SQ+U1(I)*S+U2(I))
      SUMAF=CMPLX(0.0,0.0)
      SUMBAF=CMPLX(0.0,0.0)
      SUMDAF=CMPLX(0.0,0.0)
      DO 503 I=1,4
      SUMAF=YB(I)*AF(I)+SUMAF
      SUMBAF=YPB(I)*AF(I)+SUMBAF
  503 SUMDAF=YPD(I)*AF(I)+SUMDAF
      DEN=SQ+C1
      ANUM=SUMDAF*(SQ+C1)-C2-BETA*SUMBAF-FC*SUMAF/AIXX-
     1 ALPHA*SQ-GAMMA
      WSS=(34.48)**2/(SQ+2.*(.434)*(34.48)*S+(34.48)**2)*
     1 (84.09)**2/(SQ+2.*(.594)*(84.09)*S+(84.09)**2)
      HOL = (1.0 - CEXP(-TS*S))/S
      TOTAL = -1./TS*HOL*WSS*ANUM/DEN
   24 TF =TF + TOTAL
      WR = SIN (TS*OMEG)/(1.0 + COS (TS*OMEG))
      WP = CMPLX (0.0, WR)
      ABSVAL = CABS(TF)
      DB = 20.0*ALOG10 (ABSVAL)
      PHASE = 57.29578*ATAN2(AIMAG(TF), REAL(TF))
      IF(PHASE) 30,30,31
   30 PHASE = PHASE+360.0
   31 CONTINUE
      IF (XK-1.0) 32,32,39
   39 CONTINUE
      DELT1 = ABS (DELT-PHASE)
      DALT1 = ABS (DALT-DB)
      IF (DALT1-10.0) 44,44,77
   44 IF (DELT1-15.0) 32,32,777
  777 IF (DELT1-350.0) 77,77,32
   32 DELT = PHASE
      DALT = DB
      XK = XK + 1.0
   25 FORMAT(5X,6HOMEGA=F9.5,5X,3HDB=F9.3,5X,6HPHASE=
     1 F9.3,4X,2HW=F10.6)
      PRINT 25,OMEG,DB,PHASE,WR
      EXIT=80.0
```

```
      DEL = 0.2
      OMEG = OMEG + DEL
   55 IF(OMEG-EXIT) 4,4,7
    7 CONTINUE
      STOP
      END
```

APPENDIX C

COMPENSATED NYQUIST

```
C     COMPUTATION OF PHID/BETAC SAMPLED FREQUENCY RESPONSE
C
      DIMENSION WB(4),ZETA(4),GM(4),YB(4),YPB(4),YPD(4),
     1 U1(4),U2(4),A(4),B(4)
      COMPLEX CMPLX,CEXP,S,WSS,HOL,TF,GCOMP,WP,Z,S1,SUMAF,
     1 DEN,ANUM,TOTAL,SQ,AF(4),AD,AN,SUMBAF,SUMDAF
   10 FORMAT(3(5X,E15.8))
   11 FORMAT(4(5X,E15.8))
   50 CONTINUE
      READ(5,10)C1,C2,FC
      READ(5,10)ASE,AK3,RP
      READ(5,11)ALCG,AIXX,AIE,TIME
      READ(5,11)WB(1), WB(2), WB(3), WB(4)
      READ(5,11)ZETA(1), ZETA(2), ZETA(3), ZETA(4)
      READ(5,11)GM(1), GM(2), GM(3), GM(4)
      READ(5,11)YB(1), YB(2), YB(3), YB(4)
      READ(5,11)YPB(1), YPB(2), YPB(3), YPB(4)
      READ(5,11)YPD(1), YPD(2), YPD(3), YPD(4)
      DO 9 I=1,4
    9 WB(I)=WB(I)*2.0*3.1415927
   33 FORMAT(1H1,4X,10HINPUT DATA,//)
      AK3 = AK3*57.29578
      PRINT 33
      PRINT 10,C1,C2,FC
      PRINT 10,ASE,AK3,RP
      PRINT 11,ALCG,AIXX,AIE,TIME
      PRINT 11, WB(1), WB(2), WB(3), WB(4)
      PRINT 11, ZETA(1), ZETA(2), ZETA(3), ZETA(4)
      PRINT 11, GM(1), GM(2), GM(3), GM(4)
      PRINT 11, YB(1), YB(2), YB(3), YB(4)
      PRINT 11, YPB(1), YPB(2), YPB(3), YPB(4)
      PRINT 11, YPD(1), YPD(2), YPD(3), YPD(4)
      ALPHA = (ALCG*ASE+AIE)/AIXX
      GAMMA=AK3*ASE/AIXX
      BETA=FC*ALCG/AIXX
      DO 501 I=1,4
      A(I)=RP*YB(I)/GM(I)
      B(I)=(ASE*YB(I)-AIE*YPB(I))/GM(I)
      U1(I)=2.0*ZETA(I)*WB(I)
  501 U2(I)=WB(I)**2
C     CALCULATE FREQUENCY RESPONSE
      DEL = 0.2
      XK = 1.0
   66 FORMAT(1H1,4X,30HFREQUENCY RESPONSE OUTPUT DATA ,//)
      PRINT 66
      OMEG = .005
```

```
      GO TO 4
   77 DEL = DEL/2.0
      OMEG = OMEG-DEL
    4 TF = CMPLX(0.0, 0.0)
      TS= 0.04
      OMEGS = 2.0*3.1415927/TS
      NTILT=5
      NX = 2*NTILT+1
      DO 24 J = 1, NX
      XJ = -NTILT+J-1
      OMEG1 = OMEG+XJ*OMEGS
      S = CMPLX(0.0, OMEG1)
      SQ=S**2
      DO 502 I=1,4
  502 AF(I)=(A(I)+B(I)*SQ)/(SQ+U1(I)*S+U2(I))
      SUMAF=CMPLX(0.0,0.0)
      SUMBAF=CMPLX(0.0,0.0)
      SUMDAF=CMPLX(0.0,0.0)
      DO 503 I=1,4
      SUMAF=YB(I)*AF(I)+SUMAF
      SUMBAF=YPB(I)*AF(I)+SUMBAF
  503 SUMDAF=YPD(I)*AF(I)+SUMDAF
      DEN=SQ+C1
      ANUM=SUMDAF*(SQ+C1)-C2-BETA*SUMBAF-FC*SUMAF/AIXX-
     1 ALPHA*SQ-GAMMA
      WSS=(34.48)**2/(SQ+2.*(.434)*(34.48)*S+(34.48)**2)*
     1 (84.09)**2/(SQ+2.*(.594)*(84.09)*S+(84.09)**2)
      HOL = (1.0 - CEXP(-TS*S))/S
      TOTAL = -1./TS*HOL*WSS*ANUM/DEN
   24 TF =TF + TOTAL
      WR = SIN (TS*OMEG)/(1.0 + COS (TS*OMEG))
      WP = CMPLX (0.0, WR)
      S1=CMPLX(0.0,OMEG)
      Z=CEXP(TS*S1)
      GC = 0.4
      A0 = 1.0
      A1 =-2.563
      A2 = 2.44704
      A3 =-.877786
      B1 =-2.38
      B2 = 1.8860
      B3 =-.497576
      AN = A0*Z**3+A1*Z**2+A2*Z+A3
      AD = Z**3+B1*Z**2+B2*Z+B3
      TF = TF*AN/AD*GC
      ABSVAL = CABS(TF)
      DB = 20.0*ALOG10 (ABSVAL)
      PHASE = 57.29578*ATAN2(AIMAG(TF), REAL(TF))
      IF(PHASE) 30,30,31
   30 PHASE = PHASE+360.0
   31 CONTINUE
      IF (XK-1.0) 32,32,39
```

```
 39 CONTINUE
    DELT1 = ABS (DELT-PHASE)
    DALT1 = ABS (DALT-DB)
    IF (DALT1-10.0) 44,44,77
 44 IF (DELT1-15.0) 32,32,777
777 IF (DELT1-350.0) 77,77,32
 32 DELT = PHASE
    DALT = DB
    XK = XK + 1.0
 25 FORMAT(5X,6HOMEGA=F9.5,5X,3HDB=F9.3,5X,6HPHASE=
   1 F9.3,4X,2HW=F10.6)
    PRINT 25,OMEG,DB,PHASE,WR
    EXIT=80.0
    DEL = 0.2
    OMEG = OMEG + DEL
 55 IF(OMEG-EXIT) 4,4,7
  7 CONTINUE
    STOP
    END
```

APPENDIX D

STEP RESPONSE OF THIRD-ORDER D(Z)

```
C
C     D(Z) = A0*Z**3+A1*Z**2+A2*Z+A3/(Z**3+B1*Z**2+B2*Z+B3)
C
C
      DIMENSION XP(3,2)
      DO 4 I=1,3
    4 XP(I,1)=0.0
      L=1
      UI=1.0
      A0 = 1.0
      A1 =-2.563
      A2 = 2.44704
      A3 =-.877786
      B1 =-2.38
      B2 = 1.8860
      B3 =-.497576
    2 CONTINUE
      XP(1,2)=-B1*XP(1,1)-B2*XP(2,1)-B3*XP(3,1)+UI
      XP(2,2)=XP(1,1)
      XP(3,2)=XP(2,1)
      YP=(A1-A0*B1)*XP(1,1)+(A2-A0*B2)*XP(2,1)
     1   +(A3-A0*B3)*XP(3,1)+UI*A0
      DO 1 I=1,3
    1 XP(I,1)=XP(I,2)
      J=L-1
      T=0.04*FLOAT(J)
      PRINT 3,YP,T
      L=L+1
      IF(L.LE.1001)GO TO 2
    3 FORMAT(10X,7HOUTPUT=E15.8,5X,5HTIME=E15.8)
      STOP
      END
```

APPENDIX E

ELEMENTS OF THE A, B, AND C MATRICES

A IS A 14 X 14 MATRIX

B IS A 14 X 1 MATRIX

C IS A 14 X 1 MATRIX

```
 A(1,1)=-2.*ZETA1*W1
 A(1,2)=-W1**2
 A(2,1)=1.0
 A(3,2)=W1**2
 A(3,3)=-2.*ZETA2*W2
 A(3,4)=-W2**2
 A(4,3)=1.0
 A(5,2)=(ALCG*ASE+AIE)*W2**2*(-W1**2)/AIXX
 A(5,3)=(ALCG*ASE+AIE)*W2**2*2.*ZETA2*W2/AIXX
 A(5,4)=(ALCG*ASE+AIE)*W2**2*W2**2/AIXX + W2**2*
1 (-C2-AK3*ASE/AIXX)
 A(5,6)=-C1
 A(5, 8)= -(FC*(ALCG*YPB(1)+YB(1)))/AIXX
 A(5,10)= -(FC*(ALCG*YPB(2)+YB(2)))/AIXX
 A(5,12)= -(FC*(ALCG*YPB(3)+YB(3)))/AIXX
 A(5,14)= -(FC*(ALCG*YPB(4)+YB(4)))/AIXX
 A(6,5)=1.0
 A(7,2)=(ASE*YB(1)-AIE*YPB(1))*W2**2*W1**2/GM(1)
 A(7,3)=(ASE*YB(1)-AIE*YPB(1))*W2**2*(-2.*ZETA2*W2)/GM(1)
 A(7,4)=(ASE*YB(1)-AIE*YPB(1))*W2**2*(-W2**2)/GM(1)+
1 W2**2*RP*YB(1)/GM(1)
 A(7,7)=-2.*ZETA(1)*WB(1)
 A(7,8)=-WB(1)**2
 A(8,7)=1.0
 A(9,2)=W2**2*(ASE*YB(2)-AIE*YPB(2))*W1**2/GM(2)
 A(9,3)=W2**2*(ASE*YB(2)-AIE*YPB(2))*(-2.*ZETA2*W2)/GM(2)
 A(9,4)=W2**2*(ASE*YB(2)-AIE*YPB(2))*(-W2**2)/GM(2) +
1 W2**2*RP*YB(2)/GM(2)
 A(9,9)=-2.*ZETA(2)*WB(2)
 A(9,10)=-WB(2)**2
 A(10,9)=1.0
 A(11,2)=W2**2*(ASE*YB(3)-AIE*YPB(3))/GM(3)*W1**2
 A(11,3)=W2**2*(ASE*YB(3)-AIE*YPB(3))/GM(3)*(-2.*ZETA2*W2)
 A(11,4)=W2**2*(ASE*YB(3)-AIE*YPB(3))/GM(3)*(-W2**2) +
1 W2**2*RP*YB(3)/GM(3)
 A(11,11)=-2.*ZETA(3)*WB(3)
 A(11,12)=-WB(3)**2
 A(12,11)=1.0
 A(13,2)=W2**2*(ASE*YB(4)-AIE*YPB(4))/GM(4)*W1**2
 A(13,3)=W2**2*(ASE*YB(4)-AIE*YPB(4))/GM(4)*(-2.*ZETA2*W2)
```

```
 A(13,4)=W2**2*(ASE*YB(4)-AIE*YPB(4))/GM(4)*(-W2**2) +
1 W2**2*RP*YB(4)/GM(4)
 A(13,13)=-2.*ZETA(4)*WB(4)
 A(13,14)=-WB(4)**2
 A(14,13)=1.0
```

ALL THE OTHER ELEMENTS ARE 0.0

```
 B(1)=1.0
```

ALL THE OTHER ELEMENTS ARE 0.0

```
 C(6)=1.0
 C(8)=YPD(1)
 C(10)=YPD(2)
 C(12)=YPD(3)
 C(14)=YPD(4)
```

ALL THE OTHER ELEMENTS ARE 0.0

APPENDIX F

TIME RESPONSE

```
      DIMENSION WB(4),ZETA(4),GM(4),YB(4),YPB(4),YPD(4),
     1 A(14,14),AN(14,14,50),B(14,14),C(14,14),AI(14,14),
     2 AT(14,14),AIN(14,14,50),AANT(14,14),BD(14),X(14),
     3 X1(14),CONTRL(2001)
C
C     D(Z) UNCOUPLED FROM DISCRETIZED SYSTEM TRANSITION MATRIX
C
C     QUANTIZATION EXCLUDED
C
      COMMON/COM1/XP(3,2)
      CALL TRAP
C
C     STORE INITIAL CONDITIONS ON STATES OF D(Z)
C
      DO 666 I=1,3
  666 XP(I,1)=0.0
      GC=0.4
   10 FORMAT(3(5X,E15.8))
   11 FORMAT(4(5X,E15.8))
   50 CONTINUE
      READ(5,10)C1,C2,FC
      READ(5,10)ASE,AK3,RP
      READ(5,11)ALCG,AIXX,AIE,TIME
      READ(5,11)WB(1), WB(2), WB(3), WB(4)
      READ(5,11)ZETA(1), ZETA(2), ZETA(3), ZETA(4)
      READ(5,11)GM(1), GM(2), GM(3), GM(4)
      READ(5,11)YB(1), YB(2), YB(3), YB(4)
      READ(5,11)YPB(1), YPB(2), YPB(3), YPB(4)
      READ(5,11)YPD(1), YPD(2), YPD(3), YPD(4)
      DO 9 I=1,4
    9 WB(I)=WB(I)*2.0*3.1415927
   33 FORMAT(1H1,4X,10HINPUT DATA,//)
      AK3 = AK3*57.29578
      PRINT 33
      PRINT 10,C1,C2,FC
      PRINT 10,ASE,AK3,RP
      PRINT 11,ALCG,AIXX,AIE,TIME
      PRINT 11, WB(1), WB(2), WB(3), WB(4)
      PRINT 11, ZETA(1), ZETA(2), ZETA(3), ZETA(4)
      PRINT 11, GM(1), GM(2), GM(3), GM(4)
      PRINT 11, YB(1), YB(2), YB(3), YB(4)
      PRINT 11, YPB(1), YPB(2), YPB(3), YPB(4)
      PRINT 11, YPD(1), YPD(2), YPD(3), YPD(4)
      ITER = 50
      PRINT 17,ITER
   17 FORMAT(1H1,2HN=I3,//)
```

```
      PRINT 26,TIME
   26 FORMAT(15X,12HFLIGHT TIME=F5.1,1X,7HSECONDS,//)
C
C     N IS THE ORDER OF THE SYSTEM  A  MATRIX
C
      N=14
C
C     T IS THE SAMPLING PERIOD
C
      T=.04
C
C     DEFINE ELEMENTS OF  A  MATRIX
C
      DO 1 I=1,N
      DO 1 J=1,N
    1 A(I,J)=0.0
      W1=34.48
      ZETA1=0.434
      W2=84.09
      ZETA2=0.594
      A(1,1)=-2.*ZETA1*W1
      A(1,2)=-W1**2
      DO 2 I=2,N,2
      I1=I-1
    2 A(I,I1)=1.0
      A(3,2)=W1**2
      A(3,3)=-2.*ZETA2*W2
      A(3,4)=-W2**2
      A(5,2)=(ALCG*ASE+AIE)*W2**2*(-W1**2)/AIXX
      A(5,3)=(ALCG*ASE+AIE)*W2**2*2.*ZETA2*W2/AIXX
      A(5,4)=(ALCG*ASE+AIE)*W2**2*W2**2/AIXX + W2**2*
     1 (-C2-AK3*ASE/AIXX)
      A(5,6)=-C1
      A(5, 8)= -(FC*(ALCG*YPB(1)+YB(1)))/AIXX
      A(5,10)= -(FC*(ALCG*YPB(2)+YB(2)))/AIXX
      A(5,12)= -(FC*(ALCG*YPB(3)+YB(3)))/AIXX
      A(5,14)= -(FC*(ALCG*YPB(4)+YB(4)))/AIXX
      A(7,2)=(ASE*YB(1)-AIE*YPB(1))*W2**2*W1**2/GM(1)
      A(7,3)=(ASE*YB(1)-AIE*YPB(1))*W2**2*(-2.*ZETA2*W2)/GM(1)
      A(7,4)=(ASE*YB(1)-AIE*YPB(1))*W2**2*(-W2**2)/GM(1)+
     1 W2**2*RP*YB(1)/GM(1)
      A(7,7)=-2.*ZETA(1)*WB(1)
      A(7,8)=-WB(1)**2
      A(9,2)=W2**2*(ASE*YB(2)-AIE*YPB(2))*W1**2/GM(2)
      A(9,3)=W2**2*(ASE*YB(2)-AIE*YPB(2))*(-2.*ZETA2*W2)/GM(2)
      A(9,4)=W2**2*(ASE*YB(2)-AIE*YPB(2))*(-W2**2)/GM(2) +
     1 W2**2*RP*YB(2)/GM(2)
      A(9,9)=-2.*ZETA(2)*WB(2)
      A(9,10)=-WB(2)**2
      A(11,2)=W2**2*(ASE*YB(3)-AIE*YPB(3))/GM(3)*W1**2
      A(11,3)=W2**2*(ASE*YB(3)-AIE*YPB(3))/GM(3)*(-2.*ZETA2*W2)
      A(11,4)=W2**2*(ASE*YB(3)-AIE*YPB(3))/GM(3)*(-W2**2) +
```

```
     1 W2**2*RP*YB(3)/GM(3)
      A(11,11)=-2.*ZETA(3)*WB(3)
      A(11,12)=-WB(3)**2
      A(13,2)=W2**2*(ASE*YB(4)-AIE*YPB(4))/GM(4)*W1**2
      A(13,3)=W2**2*(ASE*YB(4)-AIE*YPB(4))/GM(4)*(-2.*ZETA2*W2)
      A(13,4)=W2**2*(ASE*YB(4)-AIE*YPB(4))/GM(4)*(-W2**2) +
     1 W2**2*RP*YB(4)/GM(4)
      A(13,13)=-2.*ZETA(4)*WB(4)
      A(13,14)=-WB(4)**2
C
C     COMPUTE PHI(T), THE STATE TRANSITION MATRIX
C     ITER=NUMBER OF TERMS USED IN TAYLOR SERIES EXPANSION
C     TO CALCULATE PHI(T)
C
      DO 3 I=1,N
      DO 3 J=1,N
      AN(I,J,1)=A(I,J)*T
      B(I,J)=A(I,J)*T
    3 C(I,J)=A(I,J)*T
      DO 4 LL=2,ITER
      DO 5 I=1,N
      DO 5 J=1,N
    5 A(I,J)=C(I,J)/FLOAT(LL)
      CALL MATMUL(A,B,N,C,
      DO 6 I=1,N
      DO 6 J=1,N
    6 AN(I,J,LL)=C(I,J)
    4 CONTINUE
      DO 7 I=1,N
      DO 7 J=1,N
    7 AI(I,J)=0.0
      DO 8 I=1,N
    8 AI(I,I)=1.0
      DO 12 I=1,N
      DO 12 J=1,N
   12 AT(I,J)=AI(I,J)
      DO 15 I=1,N
      DO 15 J=1,N
      DO 15 LL=1,ITER
   15 AT(I,J)=AT(I,J)+AN(I,J,LL)
      DO 13 I=1,N
      DO 13 J=1,N
   13 PRINT 14,I,J,AT(I,J)
   14 FORMAT(5X,3HAT(,I2,1H,,I2,2H)=E20.8)
C
C     COMPUTE THE INTEGRAL OF THE STATE TRANSITION MATRIX
C
      DO 18 I=1,N
      DO 18 J=1,N
      AIN(I,J,1)=AN(I,J,1)*T/2.
      AI(I,J)=AI(I,J)*T
      B(I,J)=AN(I,J,1)
```

```
   18 C(I,J)=AIN(I,J,1)
      DO 20 LI=2,ITER
      LIL=LI+1
      DO 19 I=1,N
      DO 19 J=1,N
   19 A(I,J)=C(I,J)/FLOAT(LIL)
      CALL MATMUL(A,B,N,C)
      DO 21 I=1,N
      DO 21 J=1,N
   21 AIN(I,J,LI)=C(I,J)
   20 CONTINUE
      DO 22 I=1,N
      DO 22 J=1,N
   22 AANT(I,J)=AI(I,J)
      DO 23 I=1,N
      DO 23 J=1,N
      DO 23 LI=1,ITER
   23 AANT(I,J)=AANT(I,J)+AIN(I,J,LI)
      DO 24 I=1,N
      DO 24 J=1,N
   24 PRINT 25,I,J,AANT(I,J)
   25 FORMAT(10X,5HAANT(,I2,1H,,I2,2H)=E20.8)
C
C     BD(I) IS THE PRODUCT OF THE INTEGRATED PHI(T) MATRIX
C     AND THE B MATRIX
C
      DO 27 I=1,N
   27 BD(I)=0.0
      DO 28 I=1,N
   28 BD(I)=AANT(I,1)
      PRINT 49
   49 FORMAT(1H1)
C
C     DEFINE INITIAL CONDITIONS FOR THE SYSTEM STATES
C
      DO 29 I=1,N
   29 X(I)=0.0
      X(6)=2.0
C
C     DEFINE A DO-LOOP TO UPDATE THE SYSTEM STATES AND
C     PRINT THE OUTPUT STATES DESIRED AT EACH
C     SAMPLING INSTANT
C
      DO 999 M=1,2001
      KK=M-1
      TIME=T*FLOAT(KK)
      DO 31 I=1,N
   31 X1(I)=0.0
      DO 32 I=1,N
      DO 32 K=1,N
   32 X1(I)=X1(I)+AT(I,K)*X(K)
      PHID=X(6)+YPD(1)*X(8)+YPD(2)*X(10)+YPD(3)*X(12)+
```

```
    1 YPD(4)*X(14)
      CSBM=YPD(2)*X(10)
      CONTRL(M) = PHID*GC
      CALL DIGCOM(CONTRL(M),BETAC)
      PRINT 35,PHID,CONTRL(M),BETAC,TIME,CSBM
      DO 34 I=1,N
   34 X1(I)=X1(I)+BD(I)*BETAC
      DO 36 I=1,N
   36 X(I)=X1(I)
  999 CONTINUE
   35 FORMAT(1 X,5HPHID=E16.8,3X,7HBETAIN=E16.8,3X,6HBETAC=
     1E16.8,3X,7HDEGREES,2X,5HTIME=F6.2,3X,4HWB2=E16.8)
      GO TO 50
   16 STOP
      END
$IBFTC DIGCOM
       SUBROUTINE DIGCOM(UI,YP)
      COMMON/COM1/XP(3,2)
      A0 = 1.0
      A1 =-2.563
      A2 = 2.44704
      A3 =-.877786
      B1 =-2.38
      B2 = 1.8860
      B3 =-.497576
      XP(1,2)=-B1*XP(1,1)-B2*XP(2,1)-B3*XP(3,1)+UI
      XP(2,2)=XP(1,1)
      XP(3,2)=XP(2,1)
      YP=(A1-A0*B1)*XP(1,1)+(A2-A0*B2)*XP(2,1)
     1  +(A3-A0*B3)*XP(3,1)+UI*A0
      DO 1 I=1,3
    1 XP(I,1)=XP(I,2)
      RETURN
      END
$IBFTC MATMUL
      SUBROUTINE MATMUL(A,B,N,C)
      DIMENSION A(N,N),B(N,N),C(N,N)
C     CALCULATE C(I,J) COEFFICIENTS
      DO 3 I=1,N
      DO 3 J=1,N
      C(I,J)=0.0
      DO 4 K=1,N
    4 C(I,J)=C(I,J)+A(I,K)*B(K,J)
    3 CONTINUE
      RETURN
      END
$IBMAP TRAP
       ENTRY   TRAP
       AXT     **,4
TRAP   TRA     **
       SXA     TRAP-1,4
       CLA     8
```

```
        STA     RESET+1
        CLA     FIX
        TSX     S.SCCR,4
        STO     8
        TRA     TRAP-1
RESET   PLT     0
        TRA     **
        CLA     0
        ARS     20
        LBT
        TRA     *+2
        TRA*    RESET+1
        SXA     OUT,4
        TSX     S.WRIT,4
        PZE     3,,MES
OUT     AXT     **,4
        ZAC
        LRS     35
        TRA*    0
FIX     TRA     RESET
MES     BCI     3, **** UNDERFLOW
        END
```

APPENDIX G

QUANTIZATION

```
       SUBROUTINE DIGCOM(A,B)
      COMMON/COM1/XP(3,2)
      A0=1.0
      A1=-2625./1024.
      A2=+2506./1024.
      A3= -899./1024.
      B1=-2437./1024.
      B2=+1931./1024.
      B3= -510./1024.
      UI=1023.*A/15.
      AX=1.
      BX=1023./AX
      CALL ROUND(UI,ERR,AX,BX)
      UP=UI
      XP(1,2)=-B1*XP(1,1)-B2*XP(2,1)-B3*XP(3,1)+UP
      XP(2,2)=XP(1,1)
      XP(3,2)=XP(2,1)
      AX=2.
      BX=65535./AX
      CALL ROUND(XP(1,2),ERR,AX,BX)
      YP=(A1-A0*B1)*XP(1,1)+(A2-A0*B2)*XP(2,1)
     1  +(A3-A0*B3)*XP(3,1)+UP
      CALL ROUND(YP,ERR,AX,BX)
      DO 1 I=1,3
    1 XP(I,1)=XP(I,2)
      B=YP*15./1023.
      RETURN
      END
$IBFTC ROUND
      SUBROUTINE ROUND(A,B,AN,BN)
      X=ABS(A)
      S=A/X
      IX=X*AN
      XQ=IX
      XQ=XQ/AN
      IF(XQ-BN) 1,2,2
    1 A=S*XQ
      B=S*(X-XQ)
      RETURN
    2 A=S*BN
      B=S*(X-BN)
      RETURN
      END
```

APPENDIX H

POTENTIOMETER SETTINGS

POT	02 = .69600000	POT	04 = .69600000
POT	06 = .84100000	POT	08 = .84100000
POT	44 = .80800000	POT	45 = .23800000
POT	54 = .53364475	POT	43 = .29632090
POT	51 = .14304518	POT	15 = .06064000
POT	48 = .81778992	POT	49 = .44804253
POT	50 = .23348752	POT	17 = .01440000
POT	13 = .00617283	POT	58 = .00383412
POT	21 = .00680993	POT	19 = .03532000
POT	39 = .53883234	POT	46 = .36664618
POT	56 = .63535570	POT	05 = .63535570
POT	14 = .45470157	POT	16 = .45470157
POT	18 = .63937694	POT	22 = .63937694
POT	23 = .83810153	POT	24 = .83810153
POT	07 = .01000000	POT	60 = .01000000
POT	59 = .01000000	POT	35 = .01000000
POT	28 = .02443500	POT	40 = .10000000
POT	33 = .03585529	POT	34 = .05996379
POT	36 = .05784000	POT	37 = .00920473